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# The effect of fluid and solid properties on the auxetic behavior of porous materials having rock-like microstructures

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ABSTRACT

Materials with a negative Poisson's Ratio (PR), known as auxetics, exhibit the counterintuitive behavior of becoming wider when uniaxially stretched and thinner when compressed. Though negative PR is characteristic of polymer foams or cellular solids, tight as well as highly porous rocks have also been reported to exhibit negative PR. The paper proposes a novel auxetic structure based on pore-space configuration observed in rocks. We developed a theoretical auxetic 3D model consisting of rotating rigid bodies. To alleviate the mechanical assumption of rotating bodies, the theoretical model was modified to include crack-like features being represented by intersecting, elliptic cylinders. We then used a 3D printer to create a physical version of the modified model, whose PR was tested. We also numerically explored how the compressibility of fluids located in the pore-space of the modified model as well as how the elastic properties of the material from which the model is made of affect its auxetic behavior. We conclude that for a porous medium composed of a single material saturated with a single fluid (a) the more compliant the fluid is and (b) the lower the PR of the solid material, the lower the PR value of the composite material.



## 1. Introduction

The elastic properties of a medium determine its behavior under a given state of stress. Upon application of uniaxial stress on a given material, the negative ratio of the lateral strain to the axial strain is known as Poisson's Ratio (*PR*) (Mavko et al., 2009; Greaves et al., 2011). Isotropic materials can be defined using two elastic moduli: (a) the bulk modulus $K$, which is a measure of the material's resistance to volume deformation due to hydrostatic stress, and (b) the shear modulus $G$, which is defined as the ratio of shear stress to shear strain.

The two elastic moduli $K$ and $G$ determine the propagation speed of an incident wave traversing through a linearly elastic and isotropic medium, namely the longitudinal (P) and the transverse (S) waves. It follows that the Poisson's ratio for these materials can be described in terms of $K$ and $G$ as well as by the dynamic components, namely, P- and S-wave velocities ($V_P$ and $V_S$, respectively):

$$PR_d = \frac{3K - 2G}{2(3K + G)} = \frac{(V_P/V_S)^2 - 2}{2(V_P/V_S)^2 - 2} \quad (1)$$

where $PR_\mathrm{d}$ is the *dynamic* Poisson's ratio. In this case, $PR = PR_\mathrm{d}$, but for anisotropic media, Eq. (1) does not hold. Materials with positive *PR* become narrower in the transverse direction when uniaxially stretched. Conversely, materials with negative *PR* become wider in the transverse direction. This response to uniaxial stretching is known as *auxetic* behavior – from the Greek *auxetikos,* which means *tending to increase* Lakes, 1987; Evans, 1991; Evans et al., 1991. From Eq. (1), it follows that statistically isotropic media characterized by $G \to 0$ Pa result in $PR \to 0.5$. Conversely, media characterized by low compressive resistance, for which $K \to 0$ Pa, or alternatively highly resistant to shear deformation for which $G \gg K$, lead to $PR \to -1$. In other words, for isotropic, homogeneous, and linearly elastic materials, Poisson's ratio falls between −1 and 0.5 Greaves et al., 2011.

Explanations for the auxetic behavior of a given medium include causes that are intrinsic and extrinsic to the composition of the solid matter. On the one hand, it can be attributed to the elements or minerals composing the material whose interatomic bonds rotate and realign with deformation Li, 1976; Milstein and Huang, 1979; Jain and Verma, 1990; Baughman et al., 1998; Alderson and Evans, 2001. On the other hand, the fine microstructure of the material is such that the geometry and/or the overall distribution of the pore space (e.g., voids and cracks) of the medium creates an anti-symmetric topology, thus determining

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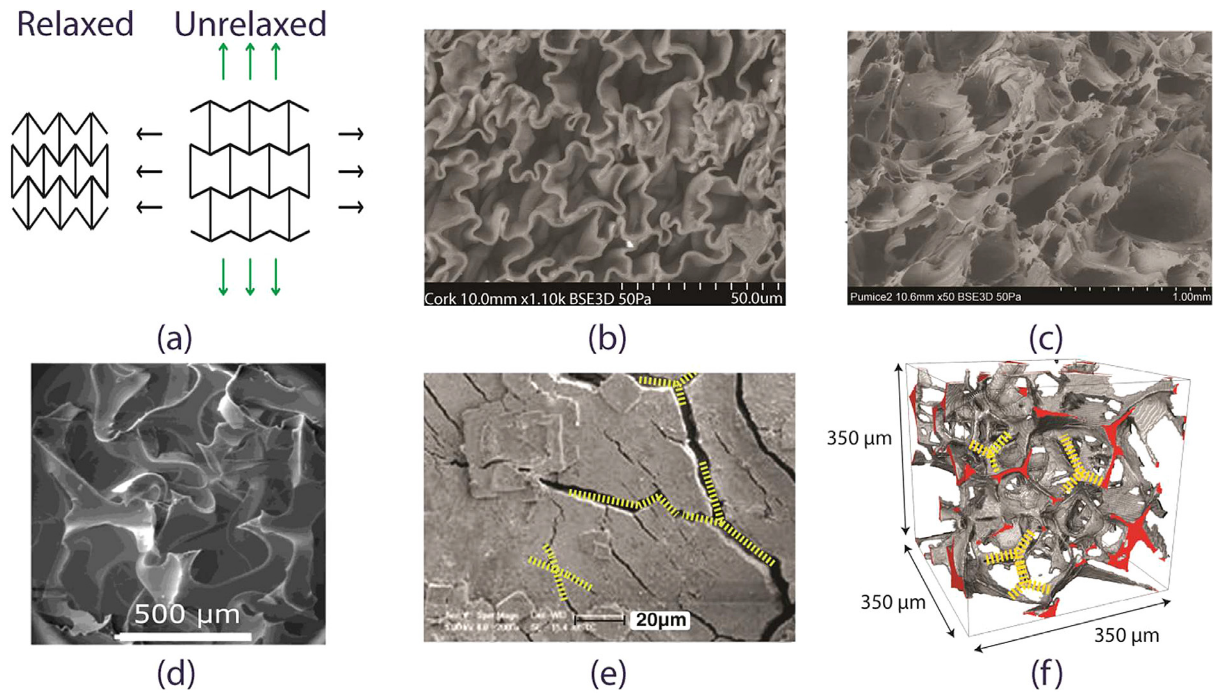


**Fig. 1.** (a) Reentrant honeycomb structure (Warren, 1990) at relaxed state (left) and upon uniaxial stretching (right), as depicted by the black arrows. The green arrows denote expansion along the transverse direction. (b) SEM image of cork's microstructure. (c) SEM image of pumice's microstructure. (d) SEM of polymeric auxetic foam as shown in Evans and Alderson (2000). (e) Image of a basalt as shown in Jiang et al. (2014). (f) Three-dimensional melt distribution of olivine-basalt aggregates as shown in Zhu et al. (2011) and Miller et al. (2014). Anti-symmetric pore configurations are emphasized using dotted yellow lines.

competitions between the way pores respond to stress — a portion of the pore space tends to close under the applied stress while the remnant, anti-symmetric distribution of pores, tends to open (Milton, 1992; Grima and Evans, 2000; Grima et al., 2000). Examples of such porous structures are normal polymer foams or cellular solids *Lakes*, 1987; Grima et al., 2006, knitted fabric Liu et al., 2010, and ceramics Tan et al., 2009, cat skin Veronda and Westmann, 1970, cow teat skin Lees et al., 1991, and cork Greaves et al., 2011. These organic and inorganic objects are mechanically considered as an open feltwork of fibers and complex network of voids, conferring the medium with an auxetic behavior. Fig. 1a exemplifies the behavior of a material exhibiting negative *PR* Gibson and Ashby, 1988; Warren, 1990 due to a specific type of microstructure — the re-entrant type. On the left, the object is at its relaxed state. On the right, upon uniaxial stretching (black arrows), the structure expands along the transverse direction, denoted with green arrows. The cork microstructure (Fig. 1b) – imaged using scanning electron microscope (SEM) – highly resembles the re-entrant (bow-tie) configuration of *Warren* (1990) (Fig. 1a).

Interestingly, other naturally-formed materials which resemble the anti-symmetric topology of the re-entrant configuration are volcanic rocks known as pumice. An SEM image of a pumice microstructure is shown in Fig. 1c, which partly resembles man-made polymeric auxetic foam shown in Fig. 1d. The pore configuration in the pumice suggests that these types of rocks may also exhibit auxetic behavior. Indeed, measurements of $V_P$ and $V_S$ on four pumice rock samples (Table 1) resulted in negative $PR_d$ values (i.e., $V_P\ /\ V_S < \sqrt{2} \approx 1.41$). Further instances of negative $PR_d$ have also been observed in tight (low porosity) rocks such as Casco Granite (*Nur and Simmons*, 1969) and Weber sandstone (*Coyner*, 1984). However, the anti-symmetric topologies in tight rocks (e.g. basalt shown in Fig. 1e) are emphasized by the pore space rather than by the mineral frame as in pumice. That is, the pore configuration of tight rocks, exhibiting intersecting crack paths of "X" and "Y" features (Fig. 1e), can be seen as the negative (i.e., inverted) microstructure of highly porous materials such as cork and pumice (Fig. 1b, c). These captivating, intersecting crack patterns can be formed by either contraction resulting from desiccation or cooling processes (*Goehring*, 2013; Goehring et al., 2015), as seen in basalts, or as viscous fluid films along grain edges dominating melt networks (Zhu et al., 2011; Miller et al., 2014). An example of the latter process of pore formation is shown in Fig. 1f through 3D visualization of pore structure that includes melt distribution of olivine-basalt aggregates.

**Table 1**
Lab measurements of P- and S-wave velocities that exhibit $V_P/V_S < \sqrt{2}$.

| Sample | $V_P$ (m/s) | $V_S$ (m/s) | *PR* | $V_P\ /\ V_S$ |
|---|---|---|---|---|
| PQ8 | 1445 | 1160 | −0.4301 | 1.24 |
| PQ3 | 2181 | 1676 | −0.2246 | 1.30 |
| PQ16A | 2547 | 1837 | −0.0529 | 1.38 |
| B1810 | 3118 | 2163 | 0.0343 | 1.44 |

In this paper, we study the effect of a pore microstructure as that observed in rocks on the elastic behavior of a material. The solid material composing such a microstructure is characterized by an isotropic, non-auxetic, phase. We propose a 3D model that has a pore network capturing the "Y-shaped" junctions and its connected forms (i.e., ≻≺ or X) that are seen in low-porosity rocks (e.g., Fig. 2a). We performed an analytical study based on the model to evaluate the relation of porosity to calculated *PR* values. We then modified the model, adding crack-like structures using elliptic cylinders and performed static numerical experiments on it to determine its directional *PR* values. As the elastic response of the porous specimen is also dependent on the elastic properties of the fluid phase that resides within the pore space and the elastic properties of the material from which it is made, we have also numerically explored these effects on the auxetic behavior of the modified model. Finally, we 3D printed our model to measure the dynamic

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