International Journal of Solids and Structures 88–89 (2016) 35–55

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Dynamic response of an infinite beam supported by a saturated poroelastic halfspace and subjected to a concentrated load moving at a constant velocity

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ARTICLE INFO

Article history:
Received 18 September 2015
Revised 22 March 2016
Available online 1 April 2016

Keywords:
Poroelasticity
Beam
Moving load
Contour integration
Bounding technique

ABSTRACT

The steady-state displacements and moments in a Bernoulli–Euler beam of finite width and infinite extent, resting on a poroelastic halfspace and subjected to a concentrated load moving at a constant velocity, were investigated using the concept of the equivalent stiffness of the halfspace. Expressions for the equivalent stiffness of the saturated poroelastic halfspace interacting with the infinite beam of finite width were derived analytically using a contour integration procedure. The influence of adhesion and drainage effects between the beam and the halfspace surface is accounted for by considering "bounding techniques" for prescribing the boundary conditions at the interface. Comparisons have been made between situations for the elastic and poroelastic halfspace with regard to their equivalent stiffness and the dynamic responses of the beam for different velocities of the moving load.

1. Introduction

The analysis of the dynamic interaction between an elastic beam and ground is fundamental to the understanding of the dynamic behavior of railway tracks with the supporting subsoil under the action of high speed trains. This topic has attracted significant research effort during the past decades because of the ever-growing public concern over the noise and vibration pollution.

Literature in rail track-ground dynamics can be divided into two categories: elastic soils and saturated poroelastic soils, according to whether the pore fluid is considered in the soil model. Filippov (1961) pioneered the research on vibrations of an infinite beam resting on an elastic halfspace subjected to a moving point load. Later, Labra (1975) investigated the effect of the axial compressive force on the critical velocity of the beam. By taking the contribution of each sleeper of the track into account, Krylov (1995, 1996) investigated the ground vibrations generated by high speed trains. Kaynia et al. (2000) developed a numerical model to predict the vibrations induced in the railway embankment and the layered elastic ground by high speed trains. Sheng et al. (1999a,b, 2003, 2004a,b) have also conducted a series of theoretical investigations on the coupled vibrations of a layered elastic ground and the rail track, which is modeled as a layered beam structure, using the analytical solutions of the wave equations . Using the same approach, more specialized models have been proposed and solved by Takemiya and Bian (2005) to include the sleeper passing effect, by Karlstrom and Bostrom (2006) to consider the rectangular embankment and by Xia et al. (2010) to investigate the contribution of the vehicle components. Numerical techniques, such as the finite element method (Hall, 2003), the boundary element method (Galvin and Dominguez, 2007; Celebi and Schmid, 2005), or a combination of them (Auersch, 2005a,b), have also been frequently employed to obtain the elastic ground vibrations due to the passage of a high speed train. For more articles on this topic, one is referred to the comprehensive review by Lombaert et al. (2015).

When there is ground water present, the pores between the soil skeleton can be completely saturated with water. A quasi-static theory of poroelasticity was developed by Biot (1941) (see also Selvadurai, 1996, 2007; Schanz, 2009 ; Cheng, 2015) and further improved by himself (Biot, 1956) to take into consideration the dynamic effects of the soil-skeleton and pore-fluid phases. Using the dynamic theory of poroelasticity (Biot, 1956), Cai et al. (2007, 2008a,b, 2010) made significant contributions to the coupled vibrations of the railway track and the saturated poroelastic halfspace under the action of a high speed train. The dynamic response of saturated ground was found to be significantly different from that of the elastic ground when the train speed increased to greater than the critical velocity of the ground.

The theoretical/numerical investigations cited above and the field experiments (Madshus and Kaynia, 2000; Lombaert et al., 2006; Lombaert and Degrande, 2009) have revealed that large

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http://dx.doi.org/10.1016/j.ijsolstr.2016.03.027
0020-7683/

dynamic amplifications appear in the track and ground as the train speed approaches an apparently critical value. This critical velocity of the track-ground system has been mathematically demonstrated by Dieterman and Metrikine (1996; 1997) using a highly-idealized model: the track structure is simplified as a Bernoulli–Euler beam with finite width and infinite extent, the ground is modeled as a homogeneous elastic halfspace and the train loading is represented by a concentrated load of constant magnitude and velocity. The equivalent stiffness of the elastic halfspace was first evaluated using the contour integration method and then substituted into the governing equations of the beam, whose responses were obtained by a numerical Fourier inversion. Two critical velocities were found to exist in this model: the first is equal to the Rayleigh wave velocity of the halfspace, at which the equivalent stiffness of the halfspace becomes zero; the second is slightly smaller, which is generated due to dynamic interaction between the beam and the halfspace. By following the same procedure, these results were extended by Kononov and Wolfert (2000) to take into account the viscous properties of the elastic halfspace. Due to the energy dissipation caused by the viscosity, the equivalent stiffness is complex at the Rayleigh wave velocity; in this case, only the second critical velocity exists.

A similar model has been employed by Jin (2004) and Xu et al. (2007). After replacing the elastic halfspace by a saturated poroelastic one, they studied the displacement responses of the beam under different load velocities. The equivalent stiffness of the poroelastic halfspace was evaluated using the numerical Fourier inversion procedure along the real axis of the wavenumber, under the assumption that no Rayleigh poles or branch points of the integrand are encountered. This assumption holds true when the soil permeability is low, since the high viscous coupling between the soil skeleton and the pore water renders the branch points and the Rayleigh pole complex-valued, and thus far away from the real axis of the wavenumber. However, when the permeability is high or in the extreme condition of infinite permeability, the Rayleigh pole and branch points would move closer to or be situated directly on the real axis of the wavenumber. In this case the summations involved in the numerical inversions may contain singularities, which would cause substantial oscillations to the resulting equivalent stiffness and thus make any evaluation of the beam response unreliable. Furthermore, the equivalent stiffness needs to be evaluated in a more rigorous fashion, for example, by using the contour integration method, so that the characteristics of the halfspace dynamics can be established mathematically.

In this study we present a mathematical formulation for the dynamic interaction problem of an infinite beam of finite width that is resting on a saturated poroelastic halfspace of infinite permeability and subjected to a concentrated load moving at a constant velocity. Four sets of boundary conditions, i.e., free draining-frictionless (*Case A*), impervious-frictionless (*Case B*), free draining-inextensible (*Case C*) and impervious-inextensible (*Case D*) boundary conditions, were prescribed over the entire surface of the poroelastic halfspace, respectively, to make the problem analytically tractable. Firstly, the equivalent stiffness of the poroelastic halfspace was derived for the four cases using the method of contour integration. Then, the equivalent stiffness is substituted into the equilibrium equation of the Bernoulli–Euler beam to obtain the displacement and moment responses using the numerical Fourier inversion. Detailed comparisons were performed for the four cases and between the elastic- and poroelastic-halfspace solutions with regard to the equivalent stiffness of the halfspace and the dynamic response of the beam for different load velocities. It is mathematically demonstrated that the first critical velocity, at which the equivalent stiffness vanishes, equals the Rayleigh wave velocity for *Cases A* and *B*, while it changes to the shear wave velocity for *Cases C* and *D*. The second critical velocity, which is due to the mechanical coupling of the beam and the halfspace, is slightly smaller than the corresponding first critical velocity in each case. The critical velocities of the beam-elastic halfspace system are found to be smaller than those of the beam-poroelastic halfspace system.

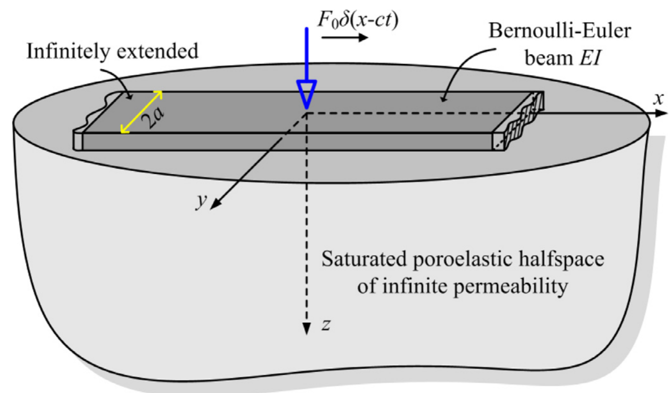

Fig. 1. Geometry of an infinite beam of finite width on a saturated poroelastic halfspace of infinite permeability.

2. Problem formulation

Fig. 1 shows the analysis model that consists of an infinite beam of finite width ($2a$) resting on the surface of a homogeneous poroelastic halfspace of infinite permeability. A moving constant load of amplitude F_0 and velocity c is applied to the center-line of the beam and acts vertical to the halfspace surface. The model is at rest initially and reaches a steady state when the load has been moving along the beam for a long time.

2.1. Governing equations

The beam experiences flexure only in the longitudinal direction and its flexural response is described by the Bernoulli–Euler beam theory

$$EI\frac{\partial^4 w_b}{\partial x^4} + m_b\frac{\partial^2 w_b}{\partial t^2} + \delta_b\frac{\partial w_b}{\partial t} + q_c(x,t) = F_0\delta(x-ct) \tag{1}$$

where w_b is the beam deflection; EI is the bending rigidity of the beam section; m_b is the mass of the beam per unit length; δ_b is the viscosity coefficient of the beam; q_c is the unknown contact line force at the beam-halfspace interface acting along the center line of the beam, which has the dimension of $[MT^{-2}]$; $\delta(\cdot)$ is the Dirac delta function and $\delta(x-ct)$ has the dimension of $[L^{-1}]$. The bending moment M_b of the beam is determined by $M_b = -EI(\partial^2 w_b/\partial x^2)$.

The dynamics of the saturated poroelastic halfspace governed by Biot's theory (Biot, 1956) take the forms

$$\mu u_{i,jj} + (\lambda + \alpha^2 M + \mu)u_{j,ji} + \alpha M w_{j,ji} = \rho\ddot{u}_i + \rho_f\ddot{w}_i \tag{2}$$

$$\alpha M u_{j,ji} + M w_{j,ji} = \rho_f\ddot{u}_i + m\ddot{w}_i + b\dot{w}_i \tag{3}$$

The constitutive equations are

$$\sigma_{ij} = \lambda\delta_{ij}\theta + \mu(u_{i,j} + u_{j,i}) - \alpha\delta_{ij}p \tag{4}$$

$$p = -\alpha M\theta + M\varsigma \tag{5}$$

where u_i and w_i ($i = x, y, z$) are the soil skeleton displacement and the pore-fluid average displacement relative to the soil skeleton, respectively; the subscripts *i, jj* and *j, ji* denote that the tensor operation and the summation convention is applied; the dots over u_i and w_i indicate the difference with respect to time *t;* λ and μ are the Lamé constants of the soil skeleton; M and α are Biot's parameter accounting for the compressibility of the two phases; they are