International Journal of Solids and Structures 88–89 (2016) 192–214

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Non-linear analysis of beam-like structures on unilateral foundations: A lattice spring model

M. Attar[a,*], A. Karrech[b], K. Regenauer-Lieb[c]

[a] School of Mechanical Engineering, The University of Western Australia, 35 Stirling Highway, Crawley, WA 6009, Australia
[b] School of Civil, Environmental and Mining Engineering, The University of Western Australia, 35 Stirling Highway, Crawley, WA 6009, Australia
[c] School of Petroleum Engineering, The University of New South Wales, 2052 Kensington, Australia

ARTICLE INFO

Article history:
Received 30 May 2015
Revised 7 March 2016
Available online 21 March 2016

Keywords:
Non-smooth dynamics
Unilateral constraint
Lattice spring model
Viscoelastic foundation
Non-linear normal modes

ABSTRACT

A computationally efficient method is presented for static and dynamic analysis of a beam-like structure on a viscoelastic foundation with the unilateral contact constraint at their interface. The non-smooth dynamics of the system is modelled using the Euler–Bernoulli theory for the beam and the bilinear Winkler model for the substrate. Thus, the unilateral contact is the only source of non-linearity. The proposed approach relies on reducing the non-smooth continuous system to a piecewise smooth multi-degree of freedom model. The beam is represented as a chain of discrete units through the use of the lattice spring model (LSM) and consequently, the phase space is divided into a number of subdomains. Hence, the system smoothness is lost when the trajectory of lattice nodes cross the boundaries between these subdomains. An effective algorithm has been developed to handle the unilateral constraints by tracking the trajectories and successively capturing the intervals that the nodes spend in each of the smooth regions. Unlike more commonly used methods, it neither relies on any prior information (such as the number and location of the lift-off areas) nor on the non-linear solvers. The accuracy and robustness of this numerical technique have been demonstrated in several application examples. Furthermore, the developed algorithm is utilised in combination with the shooting method and continuation of the periodic motions to obtain the non-linear normal modes (NNMs) of the system. We show that frequency-stiffness ($\bar{\Omega}_i^u$–$\bar{k}_w$) plots can be used to describe the salient features of the NNMs. For the beam on the full foundation, the first mode in the configuration space has a linear shape and thus, the conventional bilinear formulation can accurately approximate the non-linear frequency. For higher modes, the modal lines are open curves and the bilinear approximation loses its accuracy. In addition, internal resonances may occur once the frequencies are commensurate. They are characterised as new multimodal motions emanating from the backbone in the frequency–stiffness plot.

1. Introduction

Over the past several years, the conventional bilateral contact constraint has been widely used to assess and predict the dynamic behaviour of structural members attached to soft substrates. The classic design of piles partially embedded in soil (Prendergast et al., 2013), bridge piers (Zarafshan et al., 2012), railway tracks in high-speed transportation systems (Tran et al., 2014) and fluid-conveying pipes (Doaré, 2010) are all examples of mechanical systems with the components bonded to compliant foundations. In these conventional models, the underlying bilateral interaction manifests itself as a perfect connection between the structure and substrate. Thus, the foundation reacts equally both in compression and in tension where the restoring force is proportional to the displacement at the interface through a linear constitutive law (Fryba, 1972; Hetényi, 1946). Various mathematical frameworks, either analytical or numerical, have been previously developed and utilised in the literature in order to study the role of such a foundation in dynamic stability and vibrational behaviour, in the case of elastic semi-infinite space (Shield et al., 1994) and structure-substrate systems (Attar et al., 2014a; 2015; Mallik et al., 2006; Yokoyama, 1996). However, the main drawback of this model can be its restriction to concisely reproduce the physical reality of the interaction once the continuum tends to separate from the substrate at the interface.

In many of physical problems, the component may not be securely attached to the support or the substrate may be composed

* Corresponding author. Tel.: +61 422658830.
E-mail addresses: mostafa@mech.uwa.edu.au, mostafa.attar@gmail.com, attar_mostafa@yahoo.com (M. Attar), ali.karrech@uwa.edu.au (A. Karrech), klaus@unsw.edu.au (K. Regenauer-Lieb).

http://dx.doi.org/10.1016/j.ijsolstr.2016.03.007

of matters with different mechanical properties in compression and tension. Thus, the reaction of the support strongly depends on the direction of displacement. This can be interpreted as a bilinear non-smooth contact constraint at the interface of two bodies where the contact force is not solely a function of the system's elasticity but also the inward-outward movement directions. Note that a constraint is non-smooth if its constitutive law, which is an equation to relate restoring forces to displacement (and/or velocity) components, is non-smooth (a function $f(x)$ is smooth if it is continuously differentiable up to any order in x). Such a constraint can be found, for instance, in a submerged moored floating structure where the stiffness of the mooring structure is highly influenced by the bilinear behaviour of the tether operating in an alternating taut-slack state (Lu et al., 2013). This means that the constitutive law, which describes the tether constraining force with respect to the displacement of floating structure, is continuous but the non-smoothness originates from the discontinuity in its slope. As a result, the system response is dramatically altered, exhibiting behaviour that cannot be explained by the linear vibration theory. In particular, the modal superposition method cannot be applied to express the system's free and forced oscillations as a linear combination of independent normal modes. Therefore, there is a considerable need for more realistic models to characterise the non-smooth nature of the contact mechanism at the interface.

To this end, the so-called bimodular, bilinear, damaged, tensionless, one-way or unilateral models have been proposed (Ma et al., 2009a, b; Zhang and Murphy, 2004). The tensionless, one-way and unilateral models only react in compression, while the bilinear and bimodular models sustain compression and tension with two different stiffnesses. From the mathematical point of view, this discontinuity in the contact constitutive law is introduced by the application of Heaviside step function in the equations of motion to relate the reaction force to the sign of displacement (and/or velocity). This demonstrates the non-smoothness character of the problem, even with the assumption of the elastic behaviour of the structure in the range of small deformation theory. For instance, considering the unilateral behaviour for the Winkler foundation, which is the representation of the elastic substrate by mutually independent parallel springs, leads to an ambiguity in the system as each spring may be in either contact or non-contact states. Thus, developing a general algorithm to predict the system's response is a quite challenging task. Even a single-degree-of-freedom (SDOF) system with the unilateral constraint can lead to complex dynamics with no analogy in the underlying linear system (Shaw and Holmes, 1983; Thompson et al., 1983). This might be the main reason for the limited number of studies dealing with the non-smooth dynamics of the structural dynamic problems with the unilateral contact constraints.

The non-smooth constraints are commonly found in practical applications. Buckling and post-buckling behaviour of a rod/beam constrained inside a cylindrical/horizontal elastic constraint (Miller et al., 2015), non-linear modal analysis of riser-soil systems and corresponding vortex-induced dynamics (Neto et al., 2015), and the slackness behaviour of supporting hangers in suspension bridges (Lee and Chung, 2013) are some practical cases that can exhibit the importance of this topic. In this regard, the problem of beam-like components on the unilateral substrate is a good representative example to show that the non-smooth aspect of structural dynamic problems have seldom been noticed by the dynamics community. Consequently, there is an ongoing challenge for the establishment of a general stable solution procedure for the static analysis of such a system, not to mention the dynamic analysis (Bhattiprolu et al., 2013; Chen and Chen, 2011; Ma et al., 2011).

During mechanical deformation, the number and/or location of the lift-off zones (the areas where the component is not in contact with the foundation) will vary with respect to externally applied loads. Due to its well-known difficulty, in the most of previous studies, the information regarding detachment zones is assumed to have been already observed and provided prior to the solution. Hence, the classical theories for the contact and non-contact zones and also the compatibility conditions between them can be used for the numerical and/or analytical models describing the system in question (Coşkun, 2003; Silveira et al., 2008; Zhang and Murphy, 2004). This method is fairly computationally inexpensive and greatly simplifies the mathematical analysis. In practice, this approach reduces the problem to a set of differential equations with unknown zone lengths. Nonetheless, in reality, the range of applicability for this approach depends on the degree of complexity of the problem. In most cases, this is only limited to particular types of non-linear systems with simple geometries where it is not difficult to identify the lift-off regions before solving the problem. Obviously, the solutions based on the prior information are not flexible enough to describe the systems with more complex geometries or loading conditions, even for static analysis. For the dynamic response, the continuous matter resting on the unilateral foundation involves several contact and non-contact zones which vary with time. Thus, more efforts are needed to develop iterative algorithms in order to obtain the solution (Attar et al., 2014b).

The numerical methods for the integration of interacting bodies are theoretically convenient techniques to deal with the present unilateral contact problem without prior knowledge regarding the contact zones. At the same time, it should be noted that establishment of a robust methodology for the treatment of non-linearities in this type of interaction is not an easy task. It is obvious that the finite element analysis (FEA) and the well-known contact algorithms, e.g. penalty-based algorithms (or Lagrange multiplier method if the structure is subjected to the contact impenetrability condition) (ANSY, 2010), can be implemented to construct a full-scale contact model at the interface and then capture the unilateral constraint mechanism by employing non-linear solvers. However, due to the complexity of the tools required to perform a comprehensive investigation and the inherent mathematical issues such as stability and convergence problems, design optimisation can be infeasible for the full-scale FE model. Therefore, there has been an interest in reduced order models among scholars in recent years to reduce the computational burden of non-linear analysis for an extensive parametric study.

The objective of this study is to develop a numerically efficient scheme for the beam-foundation problems with the unilateral constraint at the interface. To develop such an approach, we use the lattice spring model (LSM Attar et al., 2014a; Griffiths and Mustoe, 2001; Pasternak and Mühlhaus, 2005) to discretise the non-smooth continuous system into a chain of discrete units and the unilateral constraints are captured by connecting the nodes to the ground through one-way spring-dampers. Accordingly, the phase space is divided into a number of regions where the system response is linear (smooth) within each region. Mathematically, the smoothness is lost when the trajectory of lattice nodes cross the borders between the subdomains. Thus, the trajectory of each node is a concatenation of smooth solutions with the transition points between them. This piecewise smooth nature of the problem is the basis for analysis of the LSM with identical nodes connected to each other through identical spring elements. A general scheme is developed to solve the governing second-order vector equation of motion for static and dynamic problems. Indeed, the solution method tracks the trajectory of the nodes and successively captures the intervals they spend in each of the smooth subdomains. Comparing the accuracy and robustness of the proposed approach versus other methods (numerical or analytical) in the example applications reveals that it is a general stable scheme with no dependency on the prior information or non-linear solvers. In addition, we investigate the salient features of the non-linear periodic motions such as