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A thermodynamically consistent plastic-damage framework for localized failure in quasi-brittle solids: Material model and strain localization analysis

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ABSTRACT

Aiming for the modeling of localized failure in quasi-brittle solids, this paper addresses a thermodynamically consistent plastic-damage framework and the corresponding strain localization analysis. A unified elastoplastic damage model is first presented based on two alternative kinematic decompositions assuming infinitesimal deformations, with the evolution laws of involved internal variables characterized by a dissipative flow tensor. For the strong (or regularized) discontinuity to form in such inelastic quasi-brittle solids and to evolve eventually into a fully softened one, a novel strain localization analysis is then suggested. A kinematic constraint more demanding than the classical discontinuous bifurcation condition is derived by accounting for the traction continuity and the loading/unloading states consistent with the kinematics of a strong (or regularized) discontinuity. More specifically, the strain jumps characterized by Maxwell's kinematic condition have to be completely inelastic (energy dissipative). Reproduction of this kinematics implies vanishing of the aforesaid dissipative flow tensorial components in the directions orthogonal to the discontinuity orientation. This property allows naturally developing a localized plastic-damage model for the discontinuity (band), with its orientation and the traction-based failure criterion consistently determined *a posteriori* from the given stress-based counterpart. The general results are then particularized to the 2D conditions of plane stress and plane strain. It is found that in the case of plane stress, strain localization into a strong (or regularized) discontinuity can occur at the onset of strain softening. Contrariwise, owing to an extra kinematic constraint, in the condition of plane strain some continuous inelastic deformations and substantial re-orientation of principal strain directions in general have to take place in the softening regime prior to strain localization. The classical Rankine, Mohr–Coulomb, von Mises (J_2) and Drucker–Prager criteria are analyzed as illustrative examples. In particular, both the closed-form solutions for the discontinuity angles validated by numerical simulations and the corresponding traction-based failure criteria are obtained.

1. Introduction

The onset of macroscopic failure in solids and structures is often signified by highly localized deformations (i.e., strain localization) within bands of small (or even fracture surfaces of negligible) width compared to the length scale of the structure in consideration. Typical examples of the manifestation of strain localization include cracks in concrete, joints in rocks, shear bands in soils, dislocations and slip lines in metals, etc., owing to the overall softening responses of these solids. It is of utmost significance to resolve strain localization and the resulting localized failure while evaluating the residual capacity and preventing the potential catastrophic collapse of structures.

Ever since the pioneering work of Ngo and Scordelis (1967) and Rashid (1968) a large number of different approaches have been developed for the modeling of localized failure in quasi-brittle solids. These approaches range from the classical discrete and smeared crack models (Rots, 1988), to the more advanced strong discontinuity approaches (Hansbo and Hansbo, 2004; Oliver, 1996; Simó et al., 1993; Wells and Sluys, 2001; Wu and Li, 2015; Wu et al., 2015). Restricting the focus to the continuum context, existing formulations can be classified into stress-based (generalized) continuum models or traction-based nonlinear fracture models. In the stress-based family the strain/displacement discontinuities

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upon strain localization are smoothed or smeared. Accordingly, the overall nonlinear behavior of the weakened solid can be described by tensorial constitutive relations in terms of stress *versus* strain equipped with internal variables. Plasticity (Chaboche, 2008; Chen, 1994) and damage mechanics (Krajcinovic, 2003; Lemaitre, 1996) or their combination (Armero and Oller, 2000; Ibrahimbegovic, 2009; Ju, 1989; Ortiz., 1985; Voyiadjis and Dorgan, 2007; Voyiadjis and Kattan, 1992; Wu et al., 2006; Zhu et al., 2010) are frequently employed to develop appropriate inelastic constitutive laws; see Abu Al-Rub and Darabi (2012) and Ibrahimbegovic et al. (2008) and the references therein. To guarantee objectivity of the energy dissipation during the failure process, the softening regime is in general regularized by introducing the fracture energy and an appropriately identified length scale (Bažant and Oh, 1983). Comparatively, in the traction-based approaches strain/displacement jumps are explicitly accounted for by embedding the discontinuities into a solid matrix along preferred orientations. It is in general assumed that energy dissipation is localized into the discontinuities while the bulk remains elastic, between which the traction continuity condition is imposed. Depending on the recoverable/irreversible properties of the discontinuities, vectorial traction-based cohesive zone models of either plastic (Carol et al., 1997), damage (Armero, 1999; Jirásek and Zimmermann, 2001) or combined plastic-damage (Wu, 2011; Wu and Xu, 2011) type can be established. Similarly, the softening law for the discontinuities is also characterized by the fracture energy.

In the traction-based modeling of localized failure in solids, a crucial step is to determine the discontinuity orientation consistently and fix it appropriately, if required. This is a non-trivial task for a new or propagating discontinuity whose orientation is not pre-defined or known *a priori*. For strain or weak discontinuities, the discontinuous bifurcation analysis, pioneered by Hill (1958; 1962), Thomas (1961) and Rice (Borré and Maier, 1989; Rice and Rudnicki, 1980; Rudnicki and Rice, 1975), nowadays becomes the standard tool. Based on the assumption of linear comparison solid (inelastic loading state in both the bulk and localization band) and the traction continuity condition, necessary conditions for the discontinuous bifurcation were identified and the orientation of shear bands can be determined for plastic materials; see the monograph (Lubarda, 2002) and the papers (Jirásek and Rolshoven, 2009; Runesson et al., 1991; Svedberg and Runesson, 1997; Voyiadjis et al., 2005; Vrech and Etse, 2005) among many others. Recently, Sánchez et al. (2008) and Huespe et al. (2009); 2012) successfully applied this strategy to the modeling of ductile fracture in presence of the stress triaxiality (Besson et al., 2003; Remmers et al., 2013).

For strong (displacement) discontinuities, similar arguments were also followed. For instance, Simó et al. (1993) and Oliver (1996) suggested using the discontinuous bifurcation condition together with *null* softening modulus to determine the discontinuity orientation. However, its application to quasi-brittle solids might be questionable, since the actual deformation states upon strain localization, i.e., inelastic loading inside the discontinuity (band) and unloading elastically outside it, are inconsistent with the assumption of linear comparison solids. Consequently, except for some particular cases (e.g., the Rankine and plane strain von Mises models), the strong discontinuity condition (Oliver, 2000; Oliver et al., 1998; 1999) cannot be satisfied in general cases (Oliver et al., 1999). Some kinematic mismatches are observed (Oliver et al., 2012; 2006a) due to mis-prediction of the discontinuity orientation, inevitably resulting in stress locking (Cervera et al., 2012; Mosler, 2005). This fact partially explains the overwhelming popularity of the maximum tensile stress criterion or linear fracture mechanics based ones (Dumstorff and Meschke, 2006) in the numerical modeling of localized failure in brittle and quasi-brittle solids (Wu and Li, 2015; Wu et al., 2015).

Provided the discontinuity orientation is determined, a cohesive zone model is generally introduced to characterize the discontinuity, resulting in either the strong/regularized or embedded/smeared discontinuity models; see Cervera and Wu (2015) for the conformity between these traction-based approaches. However, on the one hand, it is difficult to identify the traction-based failure criterion and involved parameters from available experimental data. On the other hand, the questions whether and when the traction-based cohesive zone model should be introduced cannot be easily identified. Therefore, it would be rather advantageous, if the traction-based failure criterion is derived consistently from a stress-based one and the right instant for introducing the cohesive zone model can be also identified. In this aspect, Oliver and coworkers (Oliver, 2000; Oliver et al., 1998; 1999; 2006a; 2002) made great contributions and derived cohesive zone models by projecting inelastic material laws onto the discontinuity. However, only the classical isotropic damage model (Oliver, 2000; Oliver et al., 2006a; 2002), the Rankine and plane strain von Mises plasticity models (Oliver et al., 1998; 1999) are considered. More general material constitutive laws cannot be sufficiently accounted for as declared in Oliver et al. (1999): "*Obtaining such explicit forms of the discrete constitutive equations is not so straight-forward for other families of elastoplastic models*".

Noticing the above facts, Cervera et al. (2012) proposed directly using the strong discontinuity condition (Oliver, 2000; Oliver et al., 1998; 1999) to determine the discontinuity orientation, so that the stress locking-free property can be guaranteed for a fully softened discontinuity. The discontinuity orientation for von Mises (J_2) plasticity model so obtained were validated by numerical simulations in the cases of plane stress and plane strain. Recently, the authors Wu and Cervera (2013; 2014a, 2014b; 2015) successfully extended this method to a stress-based plastic-damage model with general failure criteria. Not only the discontinuity orientation but also the traction-based cohesive zone model are determined consistently from a given stress-based inelastic material model. Furthermore, the bi-directional connections and in particular, the equivalence conditions, between two complementary methodologies for the modeling of localized failure in quasi-brittle solids, i.e., *traction-based discontinuities localized in an elastic bulk* and *strain localization of a stress-based inelastic softening solid*, have also been established. However, all our previous work assumes implicitly or explicitly that *only relative rigid body motions* occur at both sides of the discontinuity (band) upon strain localization. This restrictive kinematics implies continuous bulk strains across the discontinuity (Wu, 2011). Though the discontinuous bulk strains seldom dominate strain localization in quasi-brittle solids (Oliver et al., 2006b; Wu et al., 2015), the resulting stress continuity might be too restrictive in some cases. Moreover, the aforementioned analyses were mainly intended for the plane stress condition, and the exceptional cases which preclude occurrence of a strong (or regularized) discontinuity were not considered.

The aim of this paper is to make further contributions to the above topics. The novelties are threefold: (*i*) Maxwell's kinematic condition for guaranteeing the occurrence of a strong discontinuity is derived from the traction continuity condition together with the consistent loading/unloading deformation states upon strain localization in quasi-brittle solids; in particular, the assumption of continuous stresses across the discontinuity is disregarded; (*ii*) Closed-form results in both plane stress and plane strain conditions, coincident with those given by numerical simulations (Cervera et al., 2015), are obtained, and the consequences of an additional out-of-plane constraint in the latter case are identified; (*iii*) The aforesaid exceptional case in which the strong discontinuity is precluded for a given stress-based failure criterion is solved by introducing necessary modifications based on the equivalence between