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In-plane crushing of a hierarchical honeycomb

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ABSTRACT

Properly introduced hierarchy in cellular materials has the potential to further improve their energy absorption capacity. The in-plane uniaxial collapse response of a second order hierarchical honeycomb (i.e., a regular hexagonal honeycomb with its cell walls consisting of an equilateral triangular honeycomb) is investigated. Its failure modes for quasi-static crushing and dynamic impact in two directions are systematically explored by finite element simulations. A two-scale method is proposed and analytical expressions for the quasi-static collapse stresses of the hierarchical honeycomb in the two directions are obtained. In conjunction with the conservation of momentum, the analytical quasi-static collapse stress models are extended to dynamic crushing. The obtained theoretical collapse stresses are validated by finite element simulations for a wide range of impact velocity and relative density. Both numerical and analytical results show that the hierarchical honeycomb has an improved collapse stress over traditional hexagonal and triangular honeycombs. The improvement is found to be more pronounced for low velocity impact than for high velocity impact.

1. Introduction

Natural cellular materials such as wood and bone are in general hierarchical and have super mechanical properties (e.g., Weiner and Wagner, 1998; Gao, 2006; Katz et al., 2007; Fratzl and Weinkamer, 2007; Fleck et al., 2010). In general, the order of hierarchy can be defined as follows: a continuum solid is a zero order structure; cellular materials without hierarchical sub-structure (e.g., traditional honeycombs and foams) are considered as first order cellular materials; second order cellular materials have an additional level of structural hierarchy, i.e., the monolithic cell edges/walls of the first order structure are themselves cellular, and so on. Although many orders of hierarchy can exist in natural materials, only a few orders of hierarchy can be achieved for man-made materials, due to the limitation imposed by available manufacturing and processing techniques. To facilitate their engineering applications, a mature understanding of the mechanical properties of hierarchical structures and their constituent sub-structure is essential.

First order cellular materials have already been extensively studied and have wide applications in various fields, such as aerospace and packaging engineering, due to their promising strength, stiffness, toughness, energy absorption and so on (Gibson and Ashby, 1999; Papka and Kyriakides, 1994). For packaging applications, their impact responses are of most concern. Conventional honeycombs, as a typical first order cellular material, have been studied by a number of groups. Hönig and Stronge (2002) conducted finite element (FE) simulations of the in-plane uniaxial dynamic crushing of hexagonal honeycombs with two different wall slenderness ratios and validated the numerical results by drop weight experiments. The in-plane biaxial static and dynamic crushing responses of polycarbonate circular honeycombs were investigated numerically and experimentally by Chung and Waas (2002a, 2002b). Ruan et al. (2003) simulated the localized deformation mode of hexagonal aluminum honeycombs. The influences of the cell wall thickness and the impact velocity was considered. The dynamic crushing behavior of 2D hexagonal, distorted and Voronoi honeycombs was studied by Zheng et al. (2005) using FE method. The effects of impact velocity and cell irregularity on the deformation mode and the plateau crush stress were investigated. Zheng et al. (2014) also reported a method to identify the propagating wave front from simulated honeycombs undergoing dynamic crushing. For the dynamic crushing strength of hexagonal honeycombs, Hu and Yu (2010, 2013) derived analytical expressions for low and high velocity impacts. Their derivation was based on the corresponding microstructural collapse modes. Recently, the quasi-static and impact behaviors of both uniform and functionally graded auxetic double arrowhead honeycombs (DAHs) were explored by Qiao and Chen (2015a, 2015b). Analytical expressions for the collapse stress of uniform DAHs under low and high velocity impacts were derived and were extended for functionally graded DAHs.

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Apart from conventional honeycombs, the impact behaviors of foams have also been studied. Deshpande and Fleck (2000) investigated the strain rate sensitivity of aluminum foams using the split Hopkinson pressure bar technique. A new testing concept was proposed by Elnasri et al. (2007) for the shock enhancement of foams. The reported tests provided experimental evidence of the existence of shock front by measuring the shock stress jump as well as the shock front velocity. Kyriakides and his coworkers (Barnes et al., 2014; Gaitanaros and Kyriakides, 2014; Gaitanaros and Kyriakides, 2015) explored the crushing behavior of open-cell Al foams under different impact velocities. The high-speed photography method was used to extract the Hugoniot in terms of the shock speed vs. impact speed and the strain behind the shock vs. impact speed representations. They also generated microstructures using the *Surface Evolver* software to mimic open-cell foams and conducted three dimensional crushing simulations. The measured and calculated Hugoniot enables the calculation of the impact responses of foams without resorting to an assumed constitutive model.

In contrast to the abundant studies on first order cellular materials, only a few are available on the mechanical properties of hierarchical cellular materials. Simple recursive expressions for the stiffness and strength of hierarchical cellular solids were obtained by Lakes (1993) and Murphey and Hinkle (2003), by assuming a "continuum" model for the material on each length scale. Analytical models for the transverse stiffness and strength of second order hierarchical corrugated cores were obtained by Kooistra et al. (2007). The short wavelength failure on higher length scales was considered and several different failure modes were identified. However, the predicted strengths overestimate the experimental measurements. Two-dimensional hierarchical cellular materials made up of sandwich walls were investigated by Fan et al. (2008). Formulae for the stiffness, buckling strength, plastic collapse strength, brittle failure strength, and the fracture toughness were derived. In deriving the formulae, the contribution of the core was neglected and the enhancement in stiffness and strength was found to be due to the increased separation of the face sheets by the core. Therefore, the obtained results are upper-bound solutions. Analytical and numerical studies were conducted by Taylor et al. (2011, 2012) for the in-plane elastic properties of uniform and functionally graded hierarchical honeycombs. Various combinations of hexagonal, triangular and square honeycombs were considered for the macrostructure and substructures of hierarchical materials. Results showed that the distribution of the mass within the macrostructure has a strong effect on the Young's modulus. Simple closed-form results for all the independent elastic constants of self-similar hierarchical honeycombs with either regular square or equilateral triangular cells were obtained by Zhu et al. (2012). They found that the strength, stiffness, and natural frequency of the hierarchically nanostructured cellular materials can be tuned over a wide range. A multiscale mechanics scheme was adopted by Vigliotti and Pasini (2013) to determine the stiffness and strength of planar and three-dimensional lattices with multiple orders of structural hierarchy, showing reduced yield strength due to the presence of multiple hierarchical levels. Pugno and his coworkers (Sun and Pugno, 2013; Sun et al., 2014) proposed multifunctional hierarchical honeycombs by replacing the solid cell walls of the original regular hexagonal honeycomb with five different types of honeycombs of equal mass. The effective elastic properties of each hierarchical structure were derived using the Euler beam theory. They showed that the in-plane stiffness can be tuned by appropriately adjusting the geometrical parameters of the substructures. Vaziri and his coworkers (Ajdari et al., 2012; Haghpanah et al., 2013; Oftadeh et al., 2014) explored the stiffness and collapse strength of a regular hexagonal honeycomb with self-similar substructures. Upper-bound solutions for the plastic collapse strength of the hierarchical honeycombs were derived and validated by FE simulations.

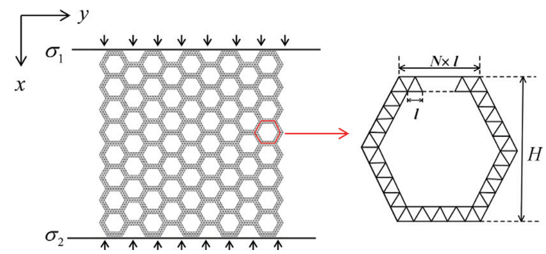

Fig. 1. Schematic of a SHH under impact loading and its unit cell.

To the best of our knowledge, the dynamic crushing response of hierarchical honeycombs has yet to be systematically explored. The purpose of this study is to investigate the quasi-static and dynamic crushing properties of a second order hierarchical honeycomb by a combination of theoretical and numerical approach. The paper is organized as follows. In Section 2, the geometrical configuration of the hierarchical honeycomb and the FE modeling are presented. In Section 3, the collapse responses of the hierarchical honeycomb subject to quasi-static uniaxial compression in the x and y directions are investigated using FE method. Analytical expressions for the collapse stress are derived, based upon the simulated deformation modes. The dynamic impact resistance of the hierarchical honeycomb under impacts is studied in Section 4. A few concluding remarks are given in Section 5.

2. Hierarchical honeycomb and finite element modeling

As discussed in the Introduction, properly introduced hierarchy in cellular materials has the potential to further improve their mechanical properties. In this study, a second order hierarchidal honeycomb (SHH) is presented and its impact responses are explored systematically. Fig. 1 shows the analysis model of the SHH subject to uniaxial compression, with a representative unit cell illustrated in the inset. As shown in Fig. 1, the macrostructure of the SHH is a regular hexagonal honeycomb, with its cell walls consisting of an equilateral triangular honeycomb (denoted by substructure). l is the edge length of substructure and $N \times l$ is the macroscopic edge length, with N being the number of substructures along the edge. The cell walls of the substructure are assumed to be uniform with thickness h. b is the out-of-plane width of the SHH. The relative density of the SHH is given by

$$\bar{\rho} = \frac{2(7N-4)}{\sqrt{3}N^2}\frac{h}{l} \quad (1)$$

FE simulations are conducted using the commercial software package ABAQUS. In the FE model, 8 × 4 cells are adopted (i.e., 8 zig-zag cells in the x direction, 4 armchair cells in the y direction). Cell walls are meshed by 4-node quadrilateral shell elements with reduced integration and large-strain formulation (S4R). Mesh sensitivity is studied. It is found that a total of 7 shell elements for each edge length of the substructure with 1 element in the out of plane direction are sufficient to ensure numerical convergence. Unless stated otherwise, $l = 0.01$ m, $b = 0.002$ m, $N = 5$ are adopted in the following FE simulations (The parameter N will be varied later). In the modeling, the SHH is placed between two rigid plates with the out-of-plane displacement being fixed. The bottom plate is constrained and a downward constant velocity is applied to the top plate to mimic impact loading. As shown in Fig. 1, the global reaction stresses of the rigid plates at the proximal and distal ends are denoted by σ_1 and σ_2, respectively. General contact is implemented to the entire model, with the tangential behavior being frictionless and the normal behavior being "Hard" contract.