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Isotoxal star-shaped polygonal voids and rigid inclusions in nonuniform antiplane shear fields. Part II: Singularities, annihilation and invisibility

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ABSTRACT

Notch stress intensity factors and stress intensity factors are obtained analytically for isotoxal star-shaped polygonal voids and rigid inclusions (and also for the corresponding limit cases of star-shaped cracks and stiffeners), when loaded through remote inhomogeneous (self-equilibrated, polynomial) antiplane shear stress in an infinite linear elastic matrix. Usually these solutions show stress singularities at the inclusion corners. It is shown that an infinite set of geometries and loading conditions exist for which not only the singularity is absent, but the stress vanishes ('annihilates') at the corners. Thus the material, which even without the inclusion corners would have a finite stress, remains unstressed at these points in spite of the applied remote load. Moreover, similar conditions are determined in which a star-shaped crack or stiffener leaves the ambient stress completely unperturbed, thus reaching a condition of 'quasi-static invisibility'. Stress annihilation and invisibility define optimal loading modes for the overall strength of a composite and are useful for designing ultra-resistant materials.

1. Introduction

The knowledge of the stress intensity factor (SIF) and of the notch stress intensity factor (NSIF), respectively, for star-shaped cracks/stiffeners and isotoxal star-shaped polygonal voids/rigid-inclusions is crucial as they represent failure criteria in the design of brittle-matrix composites (Anderson, 2005). Therefore, results presented in Part I (Dal Corso et al., 2015) of this study are complemented with the analytical, closed-form determination of SIF and NSIF. In this way, a full characterization of the stress fields near star-shaped cracks/stiffeners and polygonal voids/rigid-inclusions is reached. This allows for the analysis of the conditions of inclusion neutrality that occur when the ambient field is left unperturbed outside the inclusion. The neutrality condition has been thoroughly analyzed (Benveniste and Miloh, 2007; Bigoni et al., 1998; Mahboob and Schiavone, 2005; Ru, 1998; Ru and Schiavone, 1997; Vasudevan and Schiavone, 2005; 2006; Wang and Schiavone, 2014) because it provides a criterion for the introduction of an inclusion in a composite without a loss of strength and because it is a problem linked to homogenization techniques for composites (Christensen, 1979).

Recently, elastic cloaking in metamaterials has demonstrated wave invisibility (Brun et al., 2014; 2009; Colquitt et al., 2013; Farhat et al., 2009; Milton, 2007; Misseroni et al., 2015; Norris and Parnell, 2012) and is, in a sense, a dynamic counterpart to neutrality. Both neutrality and invisibility are strong conditions that cannot be achieved in an exact sense for a perfectly bonded inclusion (Brun et al., 2009; Ru, 1998). So neutrality, in statics, has been relaxed with the introduction of 'quasi-neutrality' (Bertoldi et al., 2007), which allows rapidly decaying stress singularities at the inclusion boundary to be neglected. In fact, considering a problem of inclusion involving singularities (as for instance at an inclusion vertex) it seems at a first glance impossible to achieve neutrality or invisibility. Nevertheless, it will be demonstrated in this paper that two special cases exist for an infinite class of geometries and modes of loading in which the 'usual' stress singularity is absent. One of these situations occurs at the vertex of a star-shaped void or rigid inclusion and will be termed 'stress annihilation', while the other, occurring for a star-shaped crack or stiffener, will be termed 'quasi-static invisibility' (or full neutrality). In the former case, the stress *vanishes* at the corner of the void/inclusion, instead of displaying the singularity which would be usually expected at a sharp corner, while, in the latter case, the star-shaped crack or star-shaped inclusion leaves the ambient stress field *completely unperturbed*, so that the inclusion becomes 'invisible' or 'fully neutral' (the word 'neutrality' is weak, because in the case analyzed in

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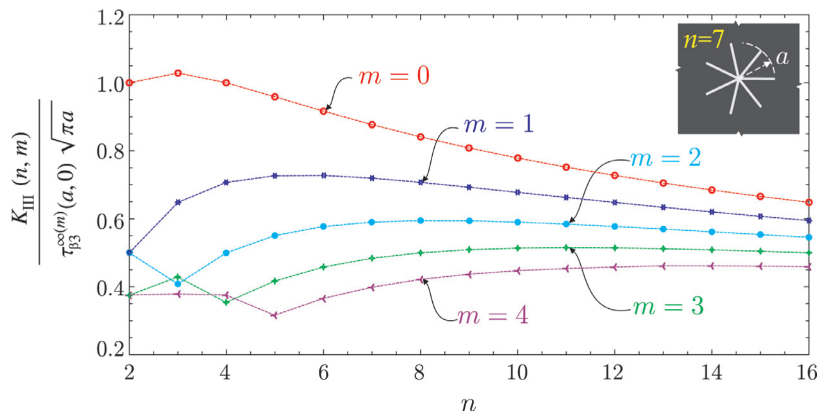

Fig. 1. Stress intensity factors for both star-shaped cracks $K_{III}^{\star}$ and star-shaped stiffeners $K_{III}^{\bigstar}$ as functions of the number n of crack or stiffener tips, for different orders m of the applied remote polynomial antiplane shear loading. Note that, due to the division by the unperturbed stress $\tau_{\beta 3}^{\infty}$ evaluated at the inclusion vertex $(a, 0)$, cracks $(\beta = 1)$ and stiffeners $(\beta = 2)$ display the same SIF, independently of the loading parameters $b_0^{(m)}$ and $c_0^{(m)}$. A single crack or stiffener corresponds to $n = 2$, which in all cases does not correspond to the maximum SIF.

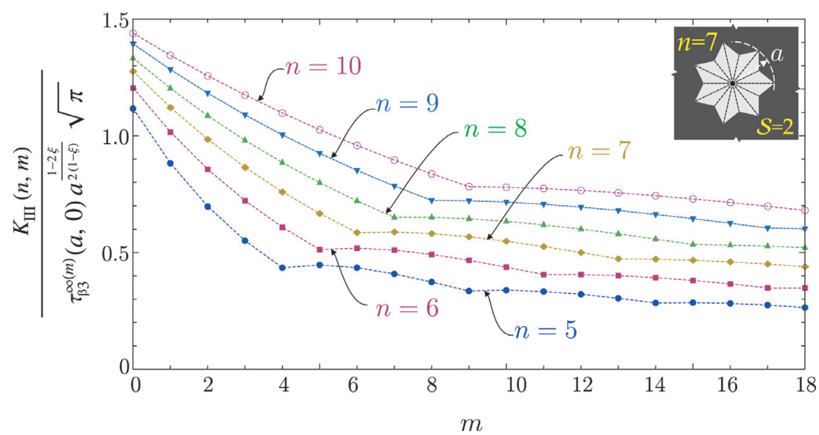

Fig. 2. Notch stress intensity factors for both isotoxal star-shaped polygonal voids $K_{III}^{\star}$ and rigid inclusions $K_{III}^{\bigstar}$ for $\mathcal{S} = 2$, functions of the order m of the applied remote polynomial antiplane shear loading. Note that, due to the division by the unperturbed stress $\tau_{\beta 3}^{\infty}$ evaluated at the inclusion vertex $(a, 0)$, voids $(\beta = 1)$ and rigid inclusions $(\beta = 2)$ display the same NSIF, independently of the loading parameters $b_0^{(m)}$ and $c_0^{(m)}$. A uniform remote shear stress field corresponds to $m = 0$, which in all cases corresponds to the maximum NSIF.

this paper the stress remains completely undisturbed everywhere in the matrix, so that the inclusion simply 'disappears'). Note that the conditions of stress annihilation and invisibility imply that the material does not fail at the void/inclusion/crack/stiffener points, but far from them and only when the material would break in the unperturbed problem. It is also shown that there are specific situations in which a partial invisibility and a partial stress annihilation is reached. In these cases invisibility or stress annihilation are verified at some but not all of the points of the star-shaped crack/stiffener or void/inclusion, so that in these cases failure of the material occurs at the points where the stress remains singular.

The results obtained in the present paper (and in Part I) refer to regular shapes of inclusions/cracks and to an infinite elastic domain, so that it is natural to address the question on how these two idealizations affect the results, particularly for quasi-static invisibility and stress annihilation. It is therefore shown that these situations can also be met for irregular star-shaped voids and cracks. Moreover, a numerical (finite elements) analysis shows that the features are present also for finite domains.

The present article is organized as follows. In Section 2 the SIFs and NSIFs, respectively for star-shaped cracks and stiffeners and for isotoxal star-shaped void and rigid inclusions will be determined. Note that the determination is in a closed-form, so that the solution does not involve infinite series. In Section 3 results will be presented in terms of stress fields around voids and cracks. Also in Section 3 the conditions for quasi-static invisibility and stress annihilation will be explained in detail, together with the situations of partial invisibility and partial stress annihilation. Generalizations to irregular star-shaped voids/inclusions (and cracks/stiffeners) and inclusions in a finite domain will be covered in Section 3.3.

2. Stress and Notch intensity factors

A measure of the stress intensification at an inclusion vertex can be obtained through the evaluation of the Stress Intensity Factor (SIF) for a star-shaped crack or a stiffener and of the Notch Stress Intensity Factor (NSIF), in the case of a polygonal void or rigid inclusion. The definition of these factors is given in relation to the specific form of remote shear stress ($\tau_{\rho 3}$ and $\tau_{\theta 3}$ in a polar coordinate system ρ, θ, and x_3), in a way that the asymptotic singular fields are represented by a constant depending only on the boundary conditions (Radaj, 2013). In particular, with reference to the decomposition (considered in Part I) of the displacement field w in its symmetric and antisymmetric parts,

$$\tau_{\rho 3}^{A}(\rho, 0) = \tau_{\vartheta 3}^{S}(\rho, 0) = 0, \tag{1}$$

the definition of SIF is introduced for star-shaped crack or stiffener

$$K_{III}^{S} = \lim_{\rho \to 0} \sqrt{2\pi\rho}\; \tau_{\rho 3}(\rho, 0), \qquad K_{III}^{A} = \lim_{\rho \to 0} \sqrt{2\pi\rho}\; \tau_{\vartheta 3}(\rho, 0), \tag{2}$$