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# A corrective solution for finding the effects of edge-rounding on complete contact between elastically similar bodies. Part II: Near-edge asymptotes and the effect of shear



R.M.N. Fleury<sup>a,\*</sup>, D.A. Hills<sup>a</sup>, J.R. Barber<sup>b</sup>

- <sup>a</sup> Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK
- <sup>b</sup> Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109-2125, USA

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#### ABSTRACT

This paper is the second part of a new approach to introduce edge rounding into elastic flat punch solutions. Building on the concept developed in the first paper of using a three-quarter plane as an asymptote to the finite punch with rounded edge problem, we now move the observation point to the very edge of the contact and introduce asymptotes for the local contact tractions so that shear may be included in the analysis to solve the partial slip problem. The edge asymptotes are used to obtain the slip zone size and the normal and tangential contact traction distribution, where the rounded edge in contact can be reasonably approximated by a half-plane.

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#### 1. Introduction

In the first part of this paper we developed a solution for the contact in the neighbourhood of the edge of a rounded quarter plane pressed onto an elastically similar half-plane (Fleury et al., 2016) (Fig. 1a). The focus of that first part was setting up of the problem and an illustration of its use in introducing slight rounding to a solution obtained under conditions of an atomically sharp edge and full adhesion. Here, we move on to look at properties near to the edge in much more detail and, in particular, introduce asymptotic forms which describe the tractions there from which conditions of local slip can be found. The asymptotic forms to the contact tractions near to the edge of the contact can be thought of as asymptotes to the asymptote, since the three-quarter plane approach is itself an asymptote to the finite punch with rounded edges problem. One of the advantages of this method is that the use of edge asymptotes enables us to correlate better local contact stresses in complex geometries, such as blade to disk assemblies, with simpler specimen geometries tested in laboratory, providing representative results (Dini et al., 2004).

The concept of edge asymptotes was first introduced by Sackfield et al. to obtain the normal pressure distribution on a rigid punch pressed onto a half-plane problem (Sackfield et al., 2003). The solution assumes that the contact problem is uncoupled, i.e.

the normal traction is not affected by the presence of shear and vice versa, which only occurs if the punch is rigid and the halfplane incompressible, or if the interface of contact is frictionless. The partial slip solution for the uncoupled problem was solved by Dini and Hills (2004), where a prediction of the extent of a slip zone from the edge of the contact was derived and the shear traction in the vicinity of the contact edge was obtained. The solution in this paper differs from that done by Banerjee et al. (2009) by using the Williams' stress intensity factors as the input to the problem, as presented in Fleury et al. (2016), instead of treating the problem as a traditional contact problem. We also expand this work to shear tractions and the partial slip contact problem which was not present in Banerjee et al. (2009).

New dimensional scaling parameters, analogous to generalized stress intensity factors, for the normal and shear tractions will be introduced and correlated with Williams' stress intensity factors,  $K_I$  and  $K_{II}$ . The square-root bounded edge asymptote to the normal contact pressure and the square-root singular asymptote to the shear traction are illustrated in Fig. 1b. Note that when the loading is proportional and no slip occurs, the shear traction will also be bounded and a square-root bounded asymptote to the shear tractions will be introduced in this case. The contact parameters are then completely characterized in the vicinity of the edge of contact in terms of these new edge scaling factors for the elastic punch with edge rounding. These parameters are introduced in the following sections and a study of the slip zone under the edge rounding is presented subsequently. In the last section of the chapter, an example problem of a finite punch pressed

<sup>\*</sup> Corresponding author. Tel.: +44 1865 273811; fax: +44 1865 273906.

E-mail addresses: rodolfo.fleury@eng.ox.ac.uk, rodolfo.nf@gmail.com (R.M.N. Fleury), david.hills@eng.ox.ac.uk (D.A. Hills), jbarber@umich.edu (J.R. Barber).

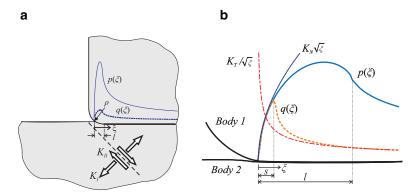


Fig. 1. (a) Three-quarter plane asymptotic model to the flat punch with rounded edges; (b) edge asymptotes to the contact tractions.

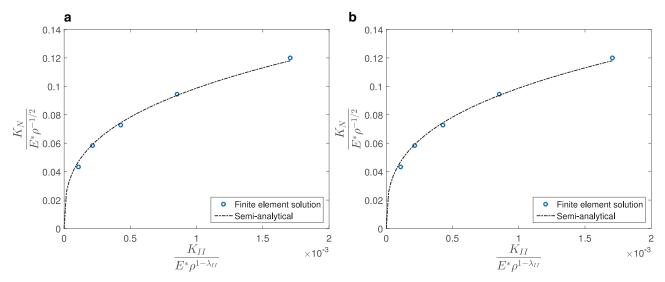


Fig. 2. Semi-analytical and FEA results of  $K_N$  as a function of (a) mode I stress intensity factor; (b) mode II stress intensity factor.

onto an elastic half-plane will be presented using the formulations derived.

#### 2. Scaling of the asymptote

One of the main reasons for embarking on this piece of work was the inadequacy of exciting asymptotic forms for a geometry of the same type deduced from half-plane theory, because such solutions display a square-root decay of tractions remote from the contact edge, and this is inconsistent with the known behavior of stress in the case of adhered, atomically sharp contact, when the observation point is much nearer to the contact edge than any other feature. On the other hand, we know that, in the neighbourhood of the very edge of contact half-plane theory does apply so, a question which presents itself is; how different is the local pressure distribution here from that derived using half-plane theory? And we have found that the best way to answer that question is to start by looking at an asymptotic form for the pressure distribution, within the asymptote, very close to the contact edge. If  $\xi$  is a coordinate measured, positive inwards, from the edge of the contact, in the rounded area we would expect to be able to describe the local pressure distribution by the leading term of a series expansion of the form

$$p(\xi) = K_N \sqrt{\xi}, \quad \xi \ll \rho, \tag{1}$$

where the multiplier,  $K_N$ , can itself be written in terms of the multipliers on the outer edge of the contact  $K_I$  and  $K_{II}$ . Note that we

restrict this solution to stress intensity factors that cause compression along the contact interface, i.e.  $K_I < 0$  and  $K_{II} > 0$ . If pure mode I or pure mode I loading is applied, the connection between the separate stress intensity factors and the local contact pressure may be written in the form

$$\left(\frac{K_N\sqrt{\rho}}{E^*}\right)^3 = H_k\left(\frac{|K_k|}{E^*\rho^{1-\lambda_k}}\right) \equiv H_k\left(|K_k^N|\right), \quad k = I, II,$$
 (2)

where  $\lambda_k$  (k=I,II) are the Williams' eigenvalues for modes I and II loading and for a right angle punch pressed onto a half-plane of similar material,  $\lambda_I=0.5445$  and  $\lambda_{II}=0.9082$ . The multipliers  $H_k$  are found from the finite element analysis as  $H_I\simeq 0.9$  and  $H_{II}\simeq 0.98^1$ . The quality of Eqs. (2) compared with the finite element solution is displayed in Fig. 2.

In part I (Fleury et al., 2016), it was shown that a trajectory in dimensionless  $K_I \times K_{II}$  space of constant contact length is also a trajectory of constant maximum peak stress and, therefore, the contact pressure in the vicinity of the rounded edge does not change in such cases. Hence, it can be reasoned that the bounded asymptote of the normal contact pressure is also constant along a trajectory of constant contact length. We can then write the relationship between applied loading and local response when both

 $<sup>^{1}</sup>$  Eq. (2) may be thought of as the first term in a series expansion, which is adequate to represent the scaling factor,  $K_{N}$ , for many practical applications. If the load is high, and the approximation to a half-plane near the contact edge is no longer valid, more terms need to be added to the series.

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