International Journal of Solids and Structures 80 (2016) 146–157

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Effective elastic moduli of fiber-reinforced composites with interfacial displacement and stress jumps

Y. Xu [a], Q.-C. He [a,b,*], S.-T. Gu [c]

[a] School of Mechanical Engineering, Southwest Jiaotong University, Chengdu 610031, China
[b] Laboratoire de Modélisation et Simulation Multi Echelle, Université Paris-Est, UMR 8208 CNRS, 5 Boulevard Descartes, 77454 Marne-la-Vallée Cedex 2, France
[c] School of Mechanics and Materials, Hohai University, Nanjing 210098, China

ARTICLE INFO

Article history:
Received 28 May 2015
Revised 14 October 2015
Available online 14 November 2015

Keywords:
Fiber-reinforced composite
Displacement jump
Traction jump
Imperfect interface
Elastic moduli

ABSTRACT

The generalized self-consistent method (GSCM) is applied to obtain the closed-form estimates for the 5 elastic moduli of a transversely isotropic composite consisting of an isotropic matrix reinforced by unidirectional isotropic fibers of circular cross-section. The interfaces between the matrix and the fibers in this composite are taken to be imperfect and characterized by the general elastic isotropic model which includes as particular cases the widely used elastic spring-layer and membrane-type imperfect interface models. The displacement, strain and stress fields in an infinite homogeneous medium containing a composite cylinder with a general imperfect interface and subjected to each of 4 elementary remote uniform loadings are specified and used in deriving the estimates for the effective elastic moduli. These results extend and bridge the special relevant results reported in the literature for fiber-reinforced composites with imperfect interfaces characterized by the spring-layer or membrane-type imperfect interface model. Numerical examples are also given to illustrate some results.

1. Introduction

Fiber-reinforced composites have been extensively studied due to their technological and theoretical importance. Within the framework of micromechanics, a large number of works have been dedicated to estimating the effective (or macroscopic) elastic moduli of a fiber-reinforced composite in terms of its fiber and matrix properties and its microstructure since the pioneering investigations (see, e.g., Hashin and Rosen, 1964; Hill, 1963, 1964; Hashin, 1979; Hashin and Shtrikman, 1963; Qiu and Weng, 1991). Accounting for the effect of imperfect fiber-matrix interfaces on the effective elastic moduli goes back to the studies of Hashin (1990). Recently, owing to the development of nanocomposites, imperfect interfaces induced by the presence of non-negligible interfacial energy have received particular attention (see, e.g., Chen and Dvorak, 2006; Chen et al., 2007; Duan et al., 2005a, 2005b, 2009; Le Quang and He, 2007a, 2007b, 2007c, 2008, 2009; Hashin, 1992).

In estimating the effective elastic moduli of fiber-reinforced composites in the context of linear elasticity, two imperfect interface models have been widely adopted. The first one is the so-called spring-layer imperfect interface model, according to which the traction vector is continuous across an interface while the displacement vector across the same interface suffers a jump proportional to the traction vector. This model was used in the works of Benveniste (1985) and Hashin (1990, 1991) and a number of other works dedicated to fiber-reinforced composites (see, e.g., Shen and Li, 2003, 2005). The second imperfect interface model is the membrane-type imperfect interface model, according to which the displacement vector across an interface is continuous whereas the traction vector across the same interface presents a jump governed by the Young–Laplace equation. The second model was first derived by Gurtin and Murdoch (1975) and has been recently widely used in studying nanocomposites (see, e.g., Chen and Dvorak, 2006; Chen et al., 2007; Duan et al., 2009; Mogilevskaya et al., 2008, 2010a, 2010b).

It is now well-established that the spring-layer and membrane-type imperfect interface models correspond to two particular cases of a general, linearly elastic, imperfect interface model derived by applying asymptotic analysis to an interphase of uniform weak thickness h between two phases so as to replace the interphase by an imperfect interface of null thickness to within an error of order $0(h^2)$ (see, e.g., Benveniste, 2006; Gu and He, 2011; Gu et al., 2014; Bövik, 1994; Hashin, 2002). Precisely, according as the elastic stiffness of the interphase is much lower or higher than that of each of the surrounding phases, the general elastic imperfect interface model degenerates into the spring-layer or membrane-type imperfect interface model.

* Corresponding author at: Laboratoire de Modélisation et Simulation Multi Echelle, Université Paris-Est, UMR 8208 CNRS, 5 Boulevard Descartes, 77454 Marne-la-Vallée Cedex 2, France. Tel.: +33 160 957 786; fax: +33 160 957 799.
E-mail address: qi-chang.he@univ-paris-est.fr (Q.-C. He).

http://dx.doi.org/10.1016/j.ijsolstr.2015.10.031
0020-7683/

Thus, the general imperfect interface model bridges the two particular extremes ones while covering the intermediate ones. In a recent work (Gu et al., 2014), the general imperfect interface model has been used in estimating the effective elastic bulk and shear moduli of isotropic particulate composites and the obtained results have extended and unified the previous relevant results reported in the literature for isotropic particulate composites in which the interfaces are characterized by the spring-layer or membrane-type imperfect interface model.

The present work, which can be viewed as a continuation of our previous one (Gu et al., 2014), aims to estimate the 5 effective elastic moduli of a transversely isotropic composite consisting of a matrix reinforced by unidirectional continuous fibers of circular cross-section. The matrix and fibers are assumed to be linearly elastic isotropic. The interfaces between the matrix and the fibers are taken to be described by the general elastic isotropic imperfect interface model whose compact formulation is provided in Gu et al. (2014). Unlike the work of Gu et al. (2014), the generalized self-consistent method (GSCM) is directly applied to estimate for the 5 effective moduli. These results are new, extending and bridging the relevant results found in the literature on fiber-reinforced composites with imperfect interfaces characterized by the spring-layer or membrane-type imperfect interface model. The results derived in the present work include as particular ones those reported in the literature on the effective elastic moduli of fiber-reinforced composites with imperfect interfaces characterized by the spring-layer and Gurtin–Murdoch models.

The paper is organized as follows. In Section 2, the local and overall constitutive relations for a transversely isotropic composite consisting of elastic isotropic fibers embedded in an elastic isotropic matrix are formulated. The general, linearly elastic, isotropic imperfect interface model is specified. In Section 3, GSCM is applied to obtain the closed-form estimates for the 5 effective elastic moduli of the composite in question. After recalling the energy consistency condition, the displacement, strain and stress fields in the fiber, matrix and effective medium involved in GSCM are explicitly provided for each of the 5 elementary macroscopic loadings. The implementation of the energy consistency condition gives rise to the closed-form estimates for the 5 effective elastic moduli. For clarity, a number of cumbersome formulae or expressions are not specified in the main text but postponed to appendices. In Section 4, numerical examples are provided so as to illustrate and discuss some results of Section 3. In Section 5, a few concluding comments are given.

2. Local, interfacial and effective constitutive relations

The composite material studied in the present work consists of a linearly elastic isotropic matrix reinforced by linearly elastic isotropic fibers aligned along one direction. For later use, we introduce a three-dimensional (3D) orthonormal basis $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ with the unit vector $\boldsymbol{e}_3$ oriented in the direction of the fibers. In the following, we refer to the matrix as phase 2 and to the fibers as phase 1. The interface between a generic fiber and the matrix is symbolized by Γ, and a unit vector $\boldsymbol{n}$ normal to Γ and oriented from the fiber to the matrix is given by

$$\boldsymbol{n} = \boldsymbol{e}_1 \cos\theta + \boldsymbol{e}_2 \sin\theta, \tag{1}$$

where θ is the angle between $\boldsymbol{n}$ and $\boldsymbol{e}_1$. Clearly, $\boldsymbol{n}$ is perpendicular to the fiber direction $\boldsymbol{e}_3$.

The local constitutive relations of the matrix and fiber phases are defined by the isotropic Hooke law:

$$\boldsymbol{\sigma}^{(i)} = \mathbb{L}^{(i)}\boldsymbol{\varepsilon}^{(i)}, \quad \mathbb{L}^{(i)} = 3k^{(i)}\left(\frac{1}{3}\boldsymbol{I}\otimes\boldsymbol{I}\right) + 2\mu^{(i)}\left(\mathbb{I} - \frac{1}{3}\boldsymbol{I}\otimes\boldsymbol{I}\right). \tag{2}$$

In this relation, the superscript i is equal to 1 or 2 according as the fiber or matrix phase is concerned; $\boldsymbol{\sigma}^{(i)}$ is the Cauchy stress tensor and $\boldsymbol{\varepsilon}^{(i)}$ is the infinitesimal strain tensor derived from the displacement vector $\boldsymbol{u}^{(i)}$ of phase i by

$$\boldsymbol{\varepsilon}^{(i)} = \frac{1}{2}\left[\nabla\boldsymbol{u}^{(i)} + \left(\nabla\boldsymbol{u}^{(i)}\right)^T\right]; \tag{3}$$

$\mathbb{L}^{(i)}$ is the elastic isotropic stiffness tensor of phase i with the bulk and shear moduli $k^{(i)}$ and $\mu^{(i)}$; $\boldsymbol{I}$ is the 3D second-order identity tensor; $\otimes$ represents the usual tensor product; $\mathbb{I}$ stands for the fourth-order identity tensor for the space of second-order symmetric tensors. Using the Kronecker tensor product $\overline{\underline{\otimes}}$ defined as

$$\left(\boldsymbol{U}\overline{\underline{\otimes}}\boldsymbol{V}\right)_{ijkl} = \left(U_{ik}V_{jl} + U_{il}V_{jk}\right)/2 \tag{4}$$

for any two second-order tensors $\boldsymbol{U}$ and $\boldsymbol{V}$, we can write $\mathbb{I} = \boldsymbol{I}\overline{\underline{\otimes}}\boldsymbol{I}$.

The interface Γ between a fiber and the matrix is taken to be imperfect. Precisely, this interface is characterized by the general linear elastic isotropic imperfect interface model derived through substituting an imperfect interface of null thickness for a linearly elastic isotropic interphase, called phase 0, of small uniform thickness h perfectly bonded to the fiber and matrix (Benveniste, 2006; Gu and He, 2011; Gu et al., 2014). To recall the formulation of this model, we first introduce the normal and tangential projection operators of second order

$$\boldsymbol{N} = \boldsymbol{n}\otimes\boldsymbol{n}, \quad \boldsymbol{T} = \boldsymbol{I} - \boldsymbol{N} \tag{5}$$

and the ones of fourth order

$$\mathbb{T} = \boldsymbol{T}\overline{\underline{\otimes}}\boldsymbol{T}, \quad \mathbb{N} = \mathbb{I} - \mathbb{T}. \tag{6}$$

Next, we define the interfacial jump operator $[[\bullet]]$, the interfacial average operator $\langle\bullet\rangle$ and the surface divergence operator $\mathrm{div}_s(\bullet)$ as follows:

$$[[\bullet]] = \bullet^{(+)} - \bullet^{(-)}, \quad \langle\bullet\rangle = \left(\bullet^{(+)} + \bullet^{(-)}\right)/2, \tag{7}$$

$$\mathrm{div}_s(\bullet) = \nabla(\bullet) : \boldsymbol{T}. \tag{8}$$

Above, $\bullet^{(+)}$ means a quantity $\bullet$ evaluated at the interface Γ on the side of the matrix while $\bullet^{(-)}$ represents a quantity $\bullet$ evaluated at the interface Γ but on the side of the fiber.

Now, we can specify the interfacial jump relations characterizing the imperfect interface Γ (Gu et al., 2014):

$$[[\boldsymbol{u}]] = \frac{h}{2}[c_1(\boldsymbol{T} : \langle\boldsymbol{\varepsilon}\rangle)\,\boldsymbol{n} + (c_2\boldsymbol{N} + c_3\boldsymbol{T})\langle\boldsymbol{t}\rangle], \tag{9}$$

$$[[\boldsymbol{t}]] = \frac{h}{2}\mathrm{div}_s[c_1(\boldsymbol{n}\cdot\langle\boldsymbol{t}\rangle)\boldsymbol{T} + (c_4\mathbb{T} + c_5\boldsymbol{T}\otimes\boldsymbol{T})\langle\boldsymbol{\varepsilon}\rangle]. \tag{10}$$

In these two relations, $\boldsymbol{t}$ is the traction vector acting on Γ given by $\boldsymbol{t} = \boldsymbol{\sigma}\boldsymbol{n}$; the 5 interfacial material parameters c_j ($j = 1, \ldots, 5$) are given in terms of the bulk and shear moduli, $k^{(i)}$ and $\mu^{(i)}$ ($i = 0, 1, 2$), of the phases and interphase by

$$\begin{aligned}
c_1 &= \frac{3k^{(2)} - 2\mu^{(2)}}{3k^{(2)} + 4\mu^{(2)}} + \frac{3k^{(1)} - 2\mu^{(1)}}{3k^{(1)} + 4\mu^{(1)}} - 2\frac{3k^{(0)} - 2\mu^{(0)}}{3k^{(0)} + 4\mu^{(0)}},\\
c_2 &= \frac{6}{3k^{(0)} + 4\mu^{(0)}} - \frac{3}{3k^{(2)} + 4\mu^{(2)}} - \frac{3}{3k^{(1)} + 4\mu^{(1)}},\\
c_3 &= \frac{2}{\mu^{(0)}} - \frac{1}{\mu^{(2)}} - \frac{1}{\mu^{(1)}},\\
c_4 &= 2\left(\mu^{(2)} + \mu^{(1)} - 2\mu^{(0)}\right),\\
c_5 &= 2\left(\frac{\mu^{(2)}\left(3k^{(2)} - 2\mu^{(2)}\right)}{3k^{(2)} + 4\mu^{(2)}} + \frac{\mu^{(1)}\left(3k^{(1)} - 2\mu^{(1)}\right)}{3k^{(1)} + 4\mu^{(1)}}\right.\\
&\quad \left. - 2\frac{\mu^{(0)}\left(3k^{(0)} - 2\mu^{(0)}\right)}{3k^{(0)} + 4\mu^{(0)}}\right).
\end{aligned} \tag{11}$$

It is useful and important to note that the imperfect interface model characterized by the relations (9) and (10) includes as two special