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Constitutive modelling of high strength titanium alloy Ti-6Al-4 V for sheet forming applications at room temperature

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ABSTRACT

To enable the design and optimisation of forming processes at room temperature the material behaviour of Ti-6Al-4 V needs to be accurately represented in numerical analysis and this requires an advanced material model. In particular, an accurate representation of the shape and size of the yield locus as well as its evolution during forming is important. In this study a rigorous set of experiments on the quasi-static deformation behaviour of a Ti-6Al-4 V alloy sheet sample at room temperature was conducted for various loading conditions and a constitutive material model developed. To quantify the anisotropy and asymmetry properties, tensile and compression tests were carried out for different specimen orientations. To examine the Bauschinger effect and the transient hardening behaviour in – plane tensile – compression and compression – tensile tests were performed. Balanced biaxial and plane strain tension tests were conducted to construct and validate the yield surface of the Ti-6Al-4 V alloy sheet sample at room temperature. A recently proposed anisotropic elastic-plastic constitutive material model, so-called HAH, was employed to describe the behaviour, in particular for load reversals. The HAH yield surface is composed of a stable component, which includes plastic anisotropy and is distorted by a fluctuating component. The key of the formulation is the use of a suitable yield function that reproduces the experimental observations well for the stable component. Meanwhile, the rapid evolution of the material structure must be captured at the macro - scale level by the fluctuating component embedded in the HAH model. Compared to conventional hardening equations, the proposed model leads to higher accuracy in predicting the Bauschinger effect and the transient hardening behaviour for the Ti-6Al-4 V sheet sample tested at room temperature.

1. Introduction

Because they combine high specific strength and high structural stiffness with excellent heat and corrosion resistance (Welsh et al., 1994), titanium alloys are increasingly used as structural material for aerospace (Peters et al., 2003, Moiseyev, 2005), petrochemical, marine (Gurrappa, 2003, Banker and Scaturro, 1997) and automotive (Schauerte and Ag, 2005) industries. Ti-6Al-4 V, denoted Ti64 in this article, is the most generally utilized Titanium alloy and records for more than half of the overall usage of titanium worldwide (Donachie, 2000). Since HCP-α is the dominating phase in Ti64 and the volume of dispersed BCC β-phase ($<10\%$) is low at room temperature, the deformation behaviour of Ti-6Al-4 V is primarily governed by the HCP-α phase, which has only a limited number of slip systems (Donachie, 2000, Gerd Lütjering, 2007).

At room temperature HCP dominated metals are known to exhibit plastic anisotropy and a pronounced tension-compression asymmetry associated with the activation of twinning. This twinning is easily activated along the c/a axis during compression, which causes the strength differential effect (SD). However, while in commercial purity titanium (CPTi) twinning concomitantly takes place with dislocation slip at room temperature, Ti64 alloys only show mechanical twinning at high strain rates and under dynamic loading. Some studies suggest that Ti64 will only mechanically twin during deformation at room temperature if it is highly alloyed with aluminium ($>$ 5.0 wt. %) and oxygen ~0.2 wt. % (Zaefferer, 2003, Conrad, 1981, Yapici et al., 2006).

Recently, Prakash et al. (2010) demonstrated that with increasing plastic deformation (moderate strains), Ti64 exhibits a change in

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texture and Lowden and Hutchinson (1975) referred this asymmetry to the differences in the hydrostatic stress level along the c- axis of the texture between tension and compression. This leads to the asymmetry of dislocation motion between the usual slip $\langle 11\bar{2}0\rangle$ and the $\langle c+a\rangle$ cross slip $\langle 11\bar{2}3\rangle$ which affects the potential for texture strengthening, and results in different texture evolutions. To accurately describe the plastic deformation of Ti64, it is important to account for those deformation mechanisms and the resulting hardening behaviour difference in tension and compression. However due to the lack of suitable macroscopic constitutive models, the classical anisotropic yield function formulated by Hill (1948), accompanied with the conventional isotropic hardening approach, is still applied to represent the behaviour of Ti64 in sheet forming simulations (Djavanroodi and Derogar, 2010). To the authors' knowledge, the development of a constitutive model capable of precisely describing the behaviour of Ti64 during load reversal has not been attempted yet. The vast majority of the previous studies focused on other HCP metals rather than Ti64 alloy sheet as extensively reviewed in (Banabic et al., 2010).

Recently, Cazacu et al., (2006) developed an anisotropic-asymmetric yield criterion, the so-called CPB06, to account for the anisotropy and the SD effect associated with twinning for pressure insensitive materials. It was reported that CPB06 is highly accurate in reproducing the yield surfaces of magnesium and CPTi (Cazacu et al., 2006). Khan et al. (2007) also applied this criterion to predict the yield locus of Ti64 and its asymmetry in yielding but reported a low level of accuracy (Khan et al., 2007). Plunkett et al. 2008 improved the capability of the CPB06 model to represent yield surface asymmetry by introducing additional linear transformation tensors operating on the Cauchy stress deviator; Namely, two linear transformations were performed for CPB06ex2 and three for CPB06ex3 (Plunkett et al., 2006). In a later study, Gilles et al. (2011) applied the orthotropic asymmetry yield criteria CPB06, CPB06ex2 and CPB06ex3 in combination with a hardening model developed by Plunkett et al. (2006), which accounts for distortional hardening due to texture evolution. With regard to describing the in-plane anisotropy of the sheet, a lower accuracy was achieved for CPB06 compared to the other two yield functions CPB06ex2 and CPB06ex3. Additionally, these yield functions led to significant differences with measured yield surfaces for Ti64 in biaxial and plane stress conditions. The authors further suggested that for an accurate description of the yield surface evolution with accumulated plastic strain, additional hardening parameters would be required. In other words, to date, there is no constitutive model allowing an accurate representation of plastic anisotropy and hardening for Ti64 at room temperature, especially if the metal strip undergoes non-monotonic deformation. To enable the widespread application of Ti64, advanced constitutive material models for the prediction of the inelastic behaviour of the material under different stress states, stress reversals and other non-proportional loading cases need to be developed (Chung and Shah, 1992).

The present paper aims at developing a novel constitutive model that is capable of representing the mechanical response of Ti64 at room temperature during load reversal. According to the literature, the asymmetry, e.g., tension-compression, in the room temperature deformation behaviour of Ti64 is insignificant at moderate strain levels and this is in contrast to CPTi in which, the asymmetry has been shown to initiate from the onset of yielding (Nixon et al., 2010, Salem et al., 2003). In Section 5 of this article, the actual yield locus of Ti64 is first experimentally determined while, in Section 6, a proposed constitutive material model is developed. Here the distortional hardening of Ti64 under multiple or continuous load changes is accounted for using the homogenous anisotropic hardening model, the so called HAH approach, proposed recently by Barlat et al. (2011). In this model, the yield surface that corresponds to the stable component of the HAH model expands around the active stress state identically to that of the isotropic hardening model for the case of monotonic loading. However, the shape of the yield surface is highly distorted away from the active stress state. During a subsequent reverse loading, the new yield stress is strongly affected by the distorted yield surface, which recovers its original shape progressively near the new stress state unless permanent softening is considered. In this manner, the asymmetry in hardening behaviour resulting from the Bauschinger effect is taken into account. This approach was recently implemented into the commercial software Abaqus implicit through the user subroutine UMAT and the FE- code formulation shown in Section 2.2. The comparison with experiments for cyclic hardening of Ti64, under different loading directions validated the model which allowed capturing the main characteristics of one deformation cycle of Ti64 in the tension compression test.

2. Elastic–plastic model of Ti-64 alloy sheet at room temperature

2.1. Theoretical framework

Recently Barlat et al. (2011) proposed the HAH model, a homogenous yield function Φ based anisotropic hardening model. The model accounts for asymmetric yielding resulting from loading path changes and pre-deformation. A brief over view of this model is given below; a description in detail can be found in (Barlat et al., 2011). The criterion is expressed as a combination of a stable component φ and the fluctuating component φ_h:

$$\Phi(\mathbf{s}) = [\varphi^q + \varphi_h^q]^{\frac{1}{q}} = \Big[\varphi^q + |f_1\,\hat{\mathbf{h}}^s : \mathbf{s} + f'_1|\hat{\mathbf{h}}^s : \mathbf{s}||^q + |f_2\hat{\mathbf{h}}^s : \mathbf{s} - \mathbf{f'_2}|\hat{\mathbf{h}}^s : \mathbf{s}||^{\mathbf{q}}\Big]^{\frac{1}{q}} = \bar{\sigma}(\boldsymbol{\sigma}) \tag{1}$$

Any regular isotropic or anisotropic homogenous yield function of first order in the form of $\Phi(\mathbf{s}) = \bar{\sigma}(\boldsymbol{\sigma})$ can be used as a stable component with the effective stress $\bar{\sigma}$. The distortion of the yield locus resulting from the loading history and the corresponding asymmetric hardening is introduced by φ_h while the exponent q controls the shape of the yield surface. The parameter φ_h depends on the stress deviator $\mathbf{s}$, the microstructure deviator $\hat{\mathbf{h}}^s$, and state variables with a particular assumption of $f_1 = f'_1$ and $f_2 = f'_2$. In Eq. (1) the microstructure deviator, $\hat{\mathbf{h}}^s$ is introduced to capture and memorize the previous deformation history. It is defined as the normalized tensor of the traceless deviator $\mathbf{h}^s$ given in Eq. (2). The constant 8/3 is introduced in Eq. (2) as a matter of convenience to describe the microstructure history deviator in a similar manner to the normalized strain rate tensor before and after the strain path change as explained in detail by Barlat et al. (2011).

$$\hat{\mathbf{h}}^s = \frac{\mathbf{h}^s}{\sqrt{\frac{8}{3}\,\mathbf{h}^s : \mathbf{h}^s}} \tag{2}$$

For the undeformed case $\mathbf{h}^s$ is equal to the stress deviator $\mathbf{s}$ (both are having the same unit, MPa) and remains constant as long as the loading path keeps unchanged. Once the loading direction is changed so that $\mathbf{h}^s \neq \mathbf{s}$, $\hat{\mathbf{h}}^s$ evolves as:

$$\frac{d\hat{\mathbf{h}}^s}{d\bar{\varepsilon}} = \begin{cases} k\Big[\hat{\boldsymbol{s}} - \frac{8}{3}\hat{\mathbf{h}}^s\Big(\hat{\boldsymbol{s}} : \hat{\mathbf{h}}^s\Big)\Big] \Rightarrow \hat{\boldsymbol{s}} : \hat{\mathbf{h}}^s \geq 0 \\ k\Big[-\hat{\boldsymbol{s}} + \frac{8}{3}\hat{\mathbf{h}}^s\Big(\hat{\boldsymbol{s}} : \hat{\mathbf{h}}^s\Big)\Big] \Rightarrow \hat{\boldsymbol{s}} : \hat{\mathbf{h}}^s < 0 \end{cases} \tag{3}$$

where $\hat{\boldsymbol{s}}$ is equivalent to $\mathbf{s}$ but normalized in the form of Eq. (2). With the new loading path $\hat{\mathbf{h}}^s$ rotates from its previous straining state towards $\hat{\boldsymbol{s}}$ by Eq. (3) at a rate controlled by k.The two variables f_1 and f_2 in Eq. (1) are functions of the two state variables g_1 and g_2, and can be expressed as

$$f_x = \left[g_x^{-q} - 1\right]^{\frac{1}{q}} \Rightarrow for\ x = 1:2 \tag{4}$$

where g_x physically represents the approximate ratio of the current flow stress to that of the hypothetical isotropic hardening flow stress,