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Statistics of dynamic fragmentation for a necking instability

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ABSTRACT

The fragmentation of a stretching rod in ductile material is investigated with the approach of Mott applied to a set of necking points. Namely, instead of randomly distributed point defects, the potential sites of fracture are the maxima of the necking instability derived from a linear perturbation analysis. The process is supposed to be dominated by the fastest growing eigenmode giving distances between potential failure sites of the order of the critical wavelength. The scatter in failure times, which is at the origin of the obscuration process controlling fragmentation, is successively linked to a probabilistic threshold for failure and the combination of several modes giving maxima of varying amplitudes. The first assumption leads to continuous fragment size distributions of the same shape as the ones obtained for a random seeding of point defects, with potential sizes limited to multiples of the critical wavelength. On the contrary, the multimodal character of the instable perturbation introduces correlations between neighboring failure points which affect the distributions significantly.

1. Introduction

The dynamic fragmentation of expanding structures (rings, cylinders, spheres or hemispheres) in ductile materials usually starts by the localization of plastic deformation in multiple necks forming potential sites of fracture. Next, the plastic strain in some of these necks rapidly increases with time up to a point when failure occurs whereas the development of other necks is arrested due to unloading waves propagating in the structure as soon as failure has started. The fragment size distribution is the result of this sequence of events which is observed on high speed camera images such as in the electromagnetic expanding ring experiments of Zhang and Ravi-Chandar (2006).

Fragmentation is modeled classically with statistical approaches postulating a random seeding of point defects (necking sites in ductile fragmentation) and a failure frequency per unit volume of the structure evolving with the mean deformation. The formation of fragments is then governed by a competition between the activation of potential failure points and the impingement of some of them. More precisely, the propagation of release waves from the points which break first leads to the development of obscured zones in which failure is inhibited. The starting point of these analyses is the work of Mott (1947) who introduced the basic elements of the statistical fragmentation theory. The fragment size distribution function was later explicited by Grady (1981) on the basis of a homogeneous nucleation process and an evolution of the cumulative area of the obscured zones which integrates the overlapping effects; the modeling of the latter is inspired by works on the kinetics of phase transformations (Johnson and Mehl, 1939). In the obscuration process, the shape of release waves strongly depends on the physics of failure in the active point defects: Mott (1947) postulated an instantaneous fracture mechanism with a stress falling to zero as soon as the point breaks. In the rigid plastic case considered by Mott, the celerity of release waves tends to infinity as the mean strain rate tends to zero; a more precise elasto-plastic treatment was later adopted by Lee (1967) and proved that the celerity was bounded by the elastic wave speed. Next, Kipp and Grady (1985) extended the obscuration analysis of Mott to unloading waves initiated by a dissipative failure process with stress decreasing continuously, which seems more relevant for ductile fracture.

Statistical fragmentation models were compared to the distribution of fragment sizes in one-dimensional experiments on ductile materials. Wesenberg and Sagartz (1977) performed a high number of explosively expanded aluminum ring tests and approached the Mott distribution as the number of tests integrated in the statistics increases. The same tendency was displayed by Zhang and Ravi-Chandar (2006, 2008) in their experiments for different materials and expanding velocities. Grady and Benson (1983) tried to fit the distribution of fragment masses from copper and aluminum electromagnetic ring expansions with the function

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obtained by Grady (1981); they managed to catch approximately the shape of their experimental plot but did not predict the formation of the largest fragments.

At the beginning of the process, localization in necks is caused by the instability of the homogeneous deformation of the structure. It is usually analyzed by linear stability analyses exhibiting the periodic instability modes. In quasi-static loading, these modes are bifurcations from the homogeneous solutions as shown by Hill and Hutchinson (1975). In dynamic loading, they are eigenmodes growing exponentially with time. These modes were identified for different geometries: thick plates in plane strain loading (Shenoy and Freund, 1999), stretching cylindrical rods (Fressengeas and Molinari, 1994; Guduru and Freund, 2002; Mercier and Molinari, 2003), expanding cylinders (Mercier and Molinari, 2004), thick plates in biaxial stretching (Jouve, 2010). These works displayed the influence of different effects such as inertia, strain hardening, thermal softening and viscoplasticity (the coupling of all effects is explicited in Jouve (2010) and Mercier et al. (2010)). In many cases, the instable modes belong to a restricted domain in wavelengths and a dominant wavelength, corresponding to the fastest growing mode, is identified. The localization pattern is supposed to result from the development of a combination of instable modes which are excited by some perturbation of the initial loading conditions. The simplest analysis only considers the fastest growing mode, the associated wavelength being the characteristic length of the localization process. A multimodal linear perturbation analysis was developed recently in El Maï et al. (2014): the statistics of spacing between maxima (necking points) was investigated at different times and proved to be fitted quite accurately by the spectrum in wavelengths of the perturbation.

Although many authors associate the onset of necking to the emergence of plastic instability (identified by the Considère (1885) criterion in uniaxial stretching for instance), the accuracy of the modal analysis to predict the position of the potential failure sites involved in the fragmentation process is still a matter of debate. Moreover, conclusions drawn from published experimental data are not all in the same way. Guduru and Freund (2002) proposed a criterion on the ratio G between the growth rate of the modes and the mean strain rate and suggested that the localization process should be governed by the first instable mode reaching a critical value G_c for this ratio. They retrieved the average neck spacing in the ring experiments of Grady and Benson (1983). With a similar approach, Jouve (2010) analyzed the expanding cylinder experiment of Olive et al. (1979): he found satisfactory agreement on both the localization time and the average neck spacing with a critical value $G_c = 10$. In an expanding hemisphere experiment with transient plain strain loading conditions near the bottom, Mercier et al. (2010) predicted the number of necks and the time and angular position for the emergence of the necking pattern. On the contrary, Zhang and Ravi-Chandar (2006) ring experiments on aluminum displayed a distribution of distances between necks with a large scatter and a maximum significantly lower than the dominant wavelength predicted by an instability analysis and the criterion of Guduru and Freund (2002). It is difficult to draw a definite conclusion from these apparently contradictory data since they may depend on many parameters such as the material tested, the geometry of the samples (ring, cylinder, sphere, ...) and the characteristics of loading.

There are at least two limits to the modal analysis. First, the instable perturbations are triggered by initial defects such as material non-homogeneities or sample imperfections and the linear analysis only holds when they can be considered as small amplitude defects. This may no longer be the case when the sample dimensions are small for instance. Another limit is that the modal analysis implicitly assumes that local defects have enough time to propagate and activate perturbation of the whole structure, which is no longer the case for very high loading rates. This issue was addressed in the work of Putelat and Triantafyllidis (2014) for an elastic ring in compression: they showed that this condition was governed by the ratio of the loading rate to the different wave celerities. At last, let us mention that when the material behavior is softening, the linear perturbation analysis exhibits instable modes with infinitely short wavelengths and therefore does not lead to a discrete set of potential failure points (the issue is investigated for plane strain loading conditions in Jouve (2013)).

However, in cases when the modal analysis is relevant, it is of real interest to give some analytical tools to connect it with the fragmentation approach. This is the aim of the present work which assumes that each maximum of a linear perturbation applied to the background homogeneous solution becomes a potential failure point when the perturbation in strain reaches a critical value. The positions and failure times of the potential failure points, pre-determined by the linear analysis, are supposed to be fixed till the end of the fragmentation process. In other words, the incidence of release waves on the background solution and, subsequently on the spectrum of instable modes, is neglected which implicitly assumes that the fragmentation phase is brief compared to the perturbation development.

Both the scatter in failure times, controlling obscuration, and the variations in neck spacing, confirmed by experimental observations, are essential features of the fragmentation process. They are obtained by considering random fluctuations of the failure strain over the structure and multimode perturbations.

In the following, the case of a cylindrical bar in tension is retained and linear stability analysis is performed in a one-dimensional framework. The eigenmodes are identified (Section 2.1). The positions and failure times of point defects associated with the maxima of an instable perturbation are investigated in two cases: a single mode perturbation and a random failure strain (Section 2.2) and a multimode perturbation and a constant failure strain (Section 2.3). In the second case, the failure times are explicited in the particular situation when the fastest growing mode is modulated by a single secondary mode with much lower growth rate. The statistics of fragment sizes are then driven from obscuration conditions deduced from the propagation of release waves emerging from these failure points. The conditions are written in Section 3.1 for an instantaneous fracture hypothesis and elastic release waves. The fragment size distributions are exhibited for a random failure strain in Section 3.2 and a bimodal perturbation in Section 3.3. The case of unloading waves associated with the Kipp and Grady (1985) linear stress release hypothesis, for which the obscuration conditions do not write as simply, is briefly discussed in Appendix A.

2. Failure points in a cylindrical bar developing necking instabilities

2.1. Linear stability analysis

A cylindrical bar of initial cross section $A_0 = \pi r_0^2$ is loaded at a constant tensile velocity at the extremities (the initial strain rate is denoted $\dot{\varepsilon}_0$). The instability analysis is performed in a one-dimensional formalism in which the triaxiality of stress in necked regions is taken into account as in Zhou et al. (2006) or Vadillo et al. (2012) for instance. The material is incompressible and strain hardening. Elasticity is supposed to be negligible in the perturbation growth phase. Thermal effects are disregarded.

Namely, all the mechanical variables are functions of the Lagrangian coordinate X along the bar and of time t. The true strain is given by:

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