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Homogenization schemes for aging linear viscoelastic matrix-inclusion composite materials with elongated inclusions

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ABSTRACT

An extension of the Mori–Tanaka and Ponte Castañeda–Willis homogenization schemes for linear elastic matrix-inclusion composites with ellipsoidal inclusions to aging linear viscoelastic composites is proposed. To do so, the method of Sanahuja (2013) dedicated to spherical inclusions is generalized to ellipsoidal inclusions under the assumption of time-independent Poisson's ratio. The obtained time-dependent strains are successfully compared to those predicted by an existing method dedicated to time-shift aging linear viscoelasticity showing the consistency of the proposed approach. Moreover, full 3D numerical simulations on complex matrix-inclusion microstructures show that the proposed scheme accurately estimates their overall time-dependent strains. Finally, it is shown that an aspect ratio of aggregates in the range 0.3–3 has no significant influence on the time-dependent strains of composites with per-phase constitutive relations representative of a real concrete.

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1. Introduction

Eshelby's solution (Eshelby, 1957) of an ellipsoidal inclusion in an elastic material has been used in various ways to upscale the behavior of composite materials. The Mori–Tanaka (Benveniste, 1987; Mori and Tanaka, 1973) scheme and the Ponte Castañeda–Willis scheme (Ponte Castañeda and Willis, 1995) have been designed to retrieve the elastic behavior of composites featuring spherical (Weng, 1984) or elongated inclusions (Tandon and Weng, 1984). Such mean-field homogenization schemes based on Eshelby's solution have been coupled to the correspondence principle (Lee, 1961; Mandel, 1966) to estimate the time-dependent strains of non-aging viscoelastic materials (Brinson and Lin, 1998; Lévesque et al., 2007; Wang and Weng, 1992): the Laplace–Carson transform turns the non-aging problem into a set of formal elastic problems in complex space. For composite materials made of elastic inclusions and a matrix modeled by a time shift method, such as many plastic materials and glasses (Odegard and Bandyopadhyay, 2011; Struik, 1978; Sullivan, 1990), the Laplace–Carson transform may still be applied in the equivalent time space (Lavergne et al., 2015a; Zheng and Weng, 2002). Yet, inverting the Laplace–Carson transform is still a compromise between accuracy and stability since this operation is ill-conditioned. This is one of the reasons why modern homogenization methods operate in the time domain (Berbenni et al., 2015; Lahellec and Suquet, 2007; Masson et al., 2012; Tran et al., 2011).

Regarding aging viscoelastic materials, a closed-form solution has been proposed by Sanahuja (2013) to handle the case of spherical inclusions in an aging linear viscoelastic matrix. Moreover, a reliable numerical procedure has been proposed to efficiently estimate the time-dependent strains. This procedure does not require inverting the Laplace–Carson transform and is able to handle any isotropic compliance.

This paper is devoted to validating and extending Sanahuja's method to ellipsoidal inclusions. Yet, the extension is limited to isotropic aging viscoelastic matrices featuring time-independent Poisson's ratio in the sense of Hilton and Yi (1998).

This extension may be valuable to study cementitious materials. Indeed, modern formulations of concrete may include aggregates (de Larrard, 1999), steel fibers, expanded polystyrene particles (Babu and Babu, 2003; Roy et al., 2005) or wood shavings (Bederina et al., 2007) as inclusions and such inclusions can change the viscoelastic properties of the material (Chern and Young, 1989).

A recent numerical study has shown that the size distribution and the shape of aggregates have little effect on the time-dependent strains of concretes made with non-aging cementitious matrices(Lavergne et al., 2015b). Full 3D numerical simulations and semi-analytical homogenization schemes delivered similar estimates of the time-dependent strains. Yet, this study was limited to polyhedral aggregates with an aspect ratio close to 1 (the aggregates were neither flat nor elongated). Consequently, there is a question left:

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does the aspect ratio of the aggregates affect the time-dependent strains of concrete made with aging cementitious matrices? To answer this question, the following steps have been carried out:

- In Section 2, the method of Sanahuja (2013) is extended to ellipsoidal inclusions. Eshelby's solution for an isotropic aging viscoelastic matrix featuring a time-independent Poisson's ratio is presented. The strain within the inclusion is still uniform and a time-dependent Eshelby's tensor may be defined. Then, the closed-form of the time-dependent localization tensor is derived using the Volterra operator. Finally, the Mori–Tanaka estimate of the overall viscoelastic behavior is obtained. The more sophisticated Ponte Castañeda–Willis linear estimate, which accounts separately for inclusion shape effects and effects of the spatial distribution of inclusion centers, is formally similarly extended to composites with elastic inclusions embedded in an aging viscoelastic matrix.
- In Section 3, the numerical procedure described in Sanahuja (2013) is used to evaluate the proposed homogenization schemes. The resulting estimates of the overall time-dependent strains are compared to existing ones for a fiber-reinforced polymer with a time-shift aging viscoelastic matrix. Then, a complex microstructure featuring 60% of polyhedral elastic aggregates embedded in an aging viscoelastic cementitious matrix is considered. The estimates of the time-dependent strains as evaluated by the method of Sanahuja (2013) and by full 3D numerical computations are first compared for spherical inclusions. Finally, the proposed extension to ellipsoidal inclusions of the Mori–Tanaka and Ponte Castañeda–Willis schemes are used to estimate the overall response of concrete-like materials. In particular, it will be shown that the aspect ratio of aggregates used in concrete does not significantly affect their viscoelastic behavior.

2. Extension of the model of Sanahuja to ellipsoidal inclusions

2.1. Estimating the overall time-dependent strains

2.1.1. Aging viscoelasticity

The stress tensor $\boldsymbol{\sigma}(t)$ in a viscoelastic material depends on the history of strain tensor $\boldsymbol{\varepsilon}(t)$. If the constitutive law is linear, the Boltzmann superposition principle states that the material properties are defined by a relaxation function (fourth order tensor), $\mathbb{C}(t,t')$, such that:

$$\boldsymbol{\sigma}(t) = \int_{-\infty}^{t} \mathbb{C}(t,t') d\boldsymbol{\varepsilon}(t')$$

where the integral is a Stieltjes integral. Similarly, the compliance function (fourth order tensor), $\mathbb{J}(t,t')$ is such that:

$$\boldsymbol{\varepsilon}(t) = \int_{-\infty}^{t} \mathbb{J}(t,t') d\boldsymbol{\sigma}(t')$$

If the elapsed time since loading is the only relevant parameter, the material is non-aging:

$$\mathbb{J}(t,t') = \lneqq(t-t')$$

However, the assumption of nonaging is not made in the following derivations. If the viscoelastic behavior is isotropic, a spherical relaxation function $K(t, t')$ and a deviatoric relaxation function $G(t, t')$ are defined, such that:

$$p(t) = \int_{-\infty}^{t} 3K(t,t') de(t')$$

$$\boldsymbol{\sigma}^d(t) = \int_{-\infty}^{t} 2G(t,t') d\boldsymbol{\varepsilon}^d(t')$$

where $e(t) = tr(\boldsymbol{\varepsilon}(t))/3,\ p(t) = tr(\boldsymbol{\sigma}(t))/3$. The tensors $\boldsymbol{\sigma}^d(t)$ and $\boldsymbol{\varepsilon}^d(t)$ are respectively the deviatoric parts of $\boldsymbol{\sigma}(t)$ and $\boldsymbol{\varepsilon}(t)$:

$$\sigma_{ij}^d(t) = \sigma_{ij}(t) - p(t)\delta_{ij}$$

$$\varepsilon_{ij}^d(t) = \varepsilon_{ij}(t) - e(t)\delta_{ij}$$

where δ_{ij} is the Kronecker symbol.

2.1.2. The homogenization method of Sanahuja

The homogenization method of Sanahuja (2013) operates in the time domain to deal with a composite featuring isotropic aging viscoelastic phases. A spherical inclusion featuring a linear viscoelastic isotropic behavior ($K_i(t, t')$, $G_i(t, t')$) is embedded in an infinite matrix featuring a linear viscoelastic isotropic behavior ($K_m(t, t')$, $G_m(t, t')$) and a strain history $\mathbf{E}(t)$ is applied far from the inclusion.

As usually for linear elasticity, the solution of this problem provides the exact solution of the localization problem of a composite made of such inclusions embedded in the matrix, in the so-called "dilute limit", i.e. for volume fractions of inclusions sufficiently low so that mechanical interactions between inclusions can be neglected. For isotropic composites, this localization solution is fully determined when purely spherical or deviatoric overall strains histories $\mathbf{E}(t)$ are considered. Sanahuja proved that the strain history in the inclusion is uniform, with value $\boldsymbol{\varepsilon}(t)$. In addition, a localization tensor $\mathbb{A}(t,t')$ is defined such that:

$$\boldsymbol{\varepsilon}(t) = \int_{-\infty}^{t} \mathbb{A}(t,\tau) : d\mathbf{E}(\tau)$$

To ease the computations, the Volterra operator has been introduced. It reads:

$$f \circ g(t,t') = \int_{-\infty}^{t} f(t,\tau) d_\tau g(\tau,t')$$

for any scalar functions f and g. The identity element of the Volterra operator H is defined from the Heaviside function:

$$(t,t') \mapsto H(t-t') = \begin{cases} 1 & \text{if } (t>t') \\ 0 & \text{if } (t<t') \end{cases}$$

The value of the H function at $t = t'$ does not need to be specified. The inverse of f in the sense of the Volterra operator is denoted as f^{-1} so that $f^{-1} \circ f = H$.

The spherical part $A_k(t, t')$ of $\mathbb{A}(t,t')$ reads:

$$A_k = (3K_i + 4G_m)^{-1} \circ (3K_m + 4G_m)$$

The closed-form expression of the deviatoric part $A_g(t, t')$ has been computed as well:

$$\begin{aligned} A_g &= H + 2(2H + 3D_m) \circ (2G_i \circ (2H + 3D_m) \\ &\quad + G_m \circ (6H - D_m))^{-1}(G_m - G_i) \end{aligned}$$

where $D_m = (K_m + G_m)^{-1} \circ \frac{2}{3}G_m$

These expressions provide functional relations between time histories of spherical and deviatoric strain prescribed far away from the inclusion and the induced uniform strain history in the latter. They can be used to extend, at least from a formal point of view, to aging viscoelasticity any linear elastic homogenization scheme based on Eshelby's solution, by simply substituting the classical tensor double contraction operations by Volterra operations. The resulting expressions will involve multiple time convolutions as well as Volterra inversions, and will thus be rather involved.

To face this difficulty, numerical procedures to compute the Volterra operator and its inverse proposed in Bažant (1972) are used to turn these formula into a practical tool (Sanahuja, 2013). The estimate of the effective behavior of a concrete will be compared to results of 3D numerical simulations in Section 3.3.1. It is to be mentioned that the method of Sanahuja produces an estimate of the effective behavior at once: the output is a matrix representing the global linear viscoelastic behavior of the composite material over an initially specified period of time. A single run of the method of Sanahuja handles all loading directions at once while two full 3D numerical computations are needed to simulate hydrostatic and shear creep tests.

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