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Elastic wave propagation in curved flexible pipes

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ABSTRACT

The elastic wave guide properties of a curved flexible pipe, idealised as a thin walled toroidal shell, are under consideration in this paper. Two mathematical models of such a shell are developed and validated. One model is analytical and is based on classical thin shell theory and the Galerkin's method is being employed. This provides an eigenvalue problem, from which the dispersion relation and the modal vectors are extracted. The other model is numerical and utilises the wave finite element method. By modelling a segment of the toroidal shell in a finite element environment, and exporting the mass and stiffness matrices, another eigenvalue problem is formulated, and the dispersion relation and the modal vectors are extracted. The two models back each other up with respect to validity and reliability. They provide insight about which waves (travelling as well as evanescent), that are supported by the toroidal shell. With this insight, it is possible to identify three regimes of wave motion, a curved beam regime, a cylinder regime, and a torus regime, and to explain the differences between these regimes. The identification of the regimes is based on analysing both dispersion diagrams and mode shapes.

1. Introduction

Flexible pipes are found in a wide range of manmade applications. Examples are risers used in the offshore industry, hydraulic hoses, vacuum cleaner hoses, and so on. Also in biology flexible pipes are found in fashion of blood vessels. Common for these examples is that the pipes are used to guide fluids. Also in many cases the pipes are submerged into a fluid, such as risers submerged in the sea, or in a more or less viscid medium, such as blood vessels suspended in the body tissue. Because the pipes under consideration here are characterised as *flexible*, the effects of fluid–structure interaction (FSI) need some sort of consideration if one of these real life systems are to be analysed and modelled. Often pipes are exposed to time varying loading. It could be structural induced vibrations from a shaking pump, or fluid induced pulsations from a beating heart. In particular the offshore industry experiences problems with flow induced vibrations in gas risers. Besides resulting in severe tonal noise pollution to the working environment these vibrations can potentially cause fatigue failure primarily at end fittings and equipment attached to the risers, cf. Goyder (2010) and Belfroid et al. (2007). In the cardiovascular system many studies indicate that the shear stress between the blood stream and the wall of the vessel plays a direct role in localisation and initiation of deadly diseases like atherosclerosis, Ma and Ng (2009) and Bessems et al. (2007). From these examples, it may be noticed that the location of the excitation and the location of where the vibrations are causing problems in general are not coinciding. Thus the wave guide properties, describing which kind of waves that potentially can travel along the pipe, must be a cornerstone when studying these piping systems.

Depending on the stiffness properties of the pipe and the excitation frequency, the wave guide properties have been successfully modelled by regarding the structure as, respectively, elastic tubular beams if the flexibility is primarily present in the axial direction and/or the excitation frequency is low, or as cylindrical shells if the cross sectional flexibility is just as important as the axial and/or the excitation frequency is higher. Disregarding FSI, both of these approaches are rather trivial, and have been described in several classical textbooks within this field.

Inclusion of FSI, on the other hand, complicates the problem. Though, with respect to the wave guide properties of elastic tubular beams with FSI, they were fully described by Padoussis (1998), whereas the wave guide properties of cylindrical shells with FSI where studied by Fuller and Fahy (1982), Pavic (1990), Sorokin et al. (2004), and many more. For fairly thick-walled pipes at low frequencies, where the beam assumption is valid, it has been found that the role of fluid is reduced to the added mass effect, see for example Søe-Knudsen and Sorokin (2010). The exception is the acoustic duct mode in a fluid inside the pipe, which, however, is not strongly affected by the curvedness of a pipe, see Rostafinski (1974). For the wave propagation in a thin elastic

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circular cylindrical shell under internal or/and external loading with a compressible fluid the main generally recognised result is the existence of 'fluid-originated' and 'structure-originated' modes. Modes of each of these two types are affected by the coupling in different amounts depending upon their shapes and excitation frequency, and this is rather well-known and understood.

In the real life pipe systems mentioned above, the evolution of the curvature is more or less random throughout the pipe with gradually changing magnitude and direction. Often this random curvedness has been approximated by sections of constant curvature. Thus, the wave guide properties, with and without FSI, of curved elastic tubular beams with constant curvature where also thoroughly studied by Padoussis (1998). Turning to the shell theory, the curved pipe with constant curvature and circular cross section can be represented by the elastic toroidal shell. In the meantime, only very few publications on the wave guide properties of the toroidal shell, either with or without FSI, are found. Though, what comes close is for example found in the papers by Jiangong et al. (2013) and by Towfighi and Kundu (2003). These are concerned with wave guide properties of so-called spherical curved plates. Specifically for the toroidal shell, the references Zhu (1992) and Zhu (1995) deals with free in-plane vibration of bounded segments of such shells in interaction with fluids. On the other hand the papers by Leung and Kwok (1994) and Ming et al. (2002) are concerned with both in-plane and out-of-plane vibrations of bounded segments, but in vacuum. In these two papers the models are developed by means of analytical methods, whereas Wang et al. (2007) and Tizzi (2015) solves similar problems by means of the finite element method. The paper by Pontrelli and Tatone (2006) is in fact concerned with wave propagation in fluid-filled toroidal shells. However, only the membrane components of the shell are imposed and the fluid is regarded as viscid and incompressible whereas the opposite assumption, where the fluid is regarded as inviscid and compressible, usually gives better results for acoustic related problems.

The ultimate goal should certainly be to assess the combined influence of fluid loading and curvedness on wave propagation in thin-walled pipes. However, this paper is concerned primarily with the assessment of the influence of curvedness on dispersion diagrams for a thin toroidal shell. It turns out that the wave guide properties of a toroidal shell without fluid loading may be acceptably described by simpler means (such as curved beam theory and cylindrical shell theory) for a range of parameters. The hypothesis is that FSI may be taken into account by similar simpler means, either as for a curved beam with FSI or as for a cylindrical shell with FSI, for the same ranges of parameters. To describe the wave guide properties by simpler means it is beneficial to classify these in accordance to their frequency-wise validity range. Typically three frequency ranges are used, the low-frequency range (LFR), the mid-frequency range (MFR), and the high-frequency range (HFR). In this paper the frequency ranges are defined as follows:

- *Low-frequency range:* In the LFR simple deterministic methods are valid. In the context of modelling flexible pipes, it means beam and rod approximations. These approximations are able to capture the most fundamental wave modes such as plane waves, torsional waves, shear waves, and bending waves.
- *Mid-frequency range:* The MFR involves more advanced methods such as shell approximations. These methods may also be valid in the LFR, but are able to capture high-order modes such as ovalisation and so on.
- *High-frequency range:* In the HFR the shortest wavelengths become equal to or less than the thickness of the shell. Hence, the shell approximations are no longer valid and 3d-elastic continuum models must be employed.

In order to demonstrate the benefits of modelling the flexible pipe as a toroidal shell, it will be compared with the above mentioned curved beam theory and cylindrical shell theory. On one hand, this comparison demonstrates the applicability of the curved beam approximation in the LFR, while it on the other hand reveals some crucial limitations in the beam prediction in the MFR. Contrary the comparison with the cylindrical shell theory shows disagreement in the LFR, whereas in the MFR range the two models turn out to merge, even if the torus is strongly curved. This insight provides important information about in which situations the waves are affected by the curvedness and in which they are not.

2. Methodology outline

To ensure both validity and reliability, two different models of the toroidal shell will be formulated. One model is analytical while the other one is numerical. The analytical model takes its origin in classical thin shell theory. By specialising the general shell equations to the geometry of the torus, a set of governing differential equations which may not have an exact solution are obtained. In lack of an exact solution an approximate solution is suggested and adapted to the problem by means of Galerkin's method. By truncating the approximate solution, Galerkin's method provides a finite eigenvalue problem where the characteristic equation defines the so-called dispersion relation while the corresponding eigenvectors represents the mode shapes. The numerical model, on the other hand, takes advantage of the finite element method. However it is not straightforward to model free waves in an infinite wave guide in standard finite element packages. Instead the so-called wave finite element (WFE) procedure described by Mace et al. (2005) is utilised. This procedure initiates with the mass and stiffness matrices generated from a finite element model of a representative finite subsection of the wave guide. These matrices could be generated by a commercial finite element package. Then, outside this finite element environment, an eigenvalue problem is formulated on these two matrices, and the wave guide properties of the infinite wave guide can be extracted.

The reliability of each model may be affected by, respectively, the truncation of the analytical solution and the number of degrees of freedom in the WFE model. Hence each model cannot stand alone, but the analytical and the numerical model must back each other up with respect to validation and reliability.

3. Problem formulation and model description

In order to isolate the impact of having a constant curvature on a flexible pipe, the problem will be stated as simple as possible. Thus the pipe will be highly idealised with the following delimitations:

- *Geometry:* Throughout the length of the pipe the cross section will be uniform and circular, the curvature of the pipe will be kept constant, and the pipe wall will be single layered and thin.
- *Material:* The material will be considered homogeneous, isotropic, linear elastic, and without damping.
- *Loadings and displacements:* The curved pipe has no pre-stress and displacements are regarded as small making membrane stiffening and other non-linearities neglectable.

3.1. The analytical approach

The thin shell theory applied in this paper is developed by Gol'denweizer and presented in the text book by Novozhilov (1959). Like many other shell theories, see Leissa (1973), this