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# A piecewise exponential model for three-dimensional analysis of sandwich panels with arbitrarily graded core

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ABSTRACT

In this paper a piecewise exponential model is proposed for analysis of three-dimensional elastic deformation of rectangular sandwich panels with graded core subjected to transverse loading. The model can be applied to any functionally graded plate or sandwich panel with graded core as long as the through thickness stiffness variation can be described by a smooth function. The new piecewise exponential model is fully validated through both comparison with results from the literature and a three-dimensional finite element method which employs user-implemented graded finite elements. As an example the new model is applied to three dimensional elasticity analysis of sandwich panels with power law variation in stiffness properties of the core and a comparative study is carried out to examine the effect of panel thickness and varying the power law index on panel's response.



## 1. Introduction

Sandwich panels or plates are three layer structures comprising two face sheets of high strength and stiffness, separated by a core of lower density and strength. When combined, these layers provide sandwich panels with high specific strength and as such they are ideally suited to a variety of engineering applications, particularly in the aerospace industry. Due to the mismatch in stiffness properties between the face sheets and the core, sandwich panels are susceptible to delamination, caused by high interfacial stresses, especially under localised loading (Abrate, 1998) or at high service temperatures (Noda, 1999).

One effective method of minimising the large interfacial shear stresses is to make use of the functionally graded material concept for the panel core. Functionally graded materials are a type of heterogeneous composite materials exhibiting gradual variation in microstructure and composition of the two constituent materials from one surface of the material to the other, resulting in properties which vary continuously across the material. Comprehensive review of research on structures incorporating functionally graded materials is given by Birman and Byrd (2007), while more recent reviews (Liew et al., 2011; Jha et al., 2013; Swaminathana et al., 2015) focused specifically on functionally graded plates.

Sandwich panels with graded core have been studied analytically, numerically and experimentally by a number of researchers. Anderson (2003) developed a 3D elasticity solution for a sandwich panel with orthotropic face sheets and an isotropic functionally graded core subjected to transverse loading by a rigid sphere. Kirugulige et al. (2005) demonstrated feasibility of using functionally graded foam as core material in sandwich panels under impact loading conditions using graded core with a bilinear volume fraction variation. Zhu and Sankar (2007) studied sandwich panels with graded foam core used in thermal protection systems subjected to transverse loads. The Fourier analysis combined with the Galerkin method was used for solving the two-dimensional elasticity equations and analyse panels with arbitrary variation of thermo-mechanical properties in the thickness direction. The analysis was also performed using sandwich plate theory. Significant differences were found in the results suggesting that the sandwich theory may not be suitable for the analysis of thick sandwich panels. Apetre et al. (2008) investigated several available sandwich beam theories for their suitability to analysis of sandwich plates with functionally graded core. Kashtalyan and Menshykova (2009) developed a three dimensional elasticity solution for sandwich panels with functionally graded core whose shear moduli vary exponentially through the thickness of the core. Etemadi et al. (2009) performed a finite element analysis of low velocity impact behaviour of sandwich beams with a functionally graded core. Projectile velocity and beam geometry were varied and comparisons were made between a functionally graded and a homogeneous beam. The results showed that insertion of a graded core decreased the maximum strains, yet increased maximum contact force. Rahmani et al. (2009) analysed free vibrations of sandwich

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panels with graded syntactic core using a higher-order sandwich beam theory. Icardi and Ferrero (2009) carried out optimisation studies on sandwich panels to find the orientation of the reinforcement in the face sheets and the variation of the core properties across the thickness with the view to minimise the interlaminar stress concentration under point loading. They employed an approach based a refined zig-zag model with a high-order variation of displacements. Dynamic response of sandwich panels with E-glass/Vinyl Ester face sheets and graded styrene foam cores to shock wave loading was experimentally investigated by Wang et al. (2009). Woodward and Kashtalyan (2010, 2011, 2012) investigated three-dimensional elastic deformation of sandwich panels with exponentially graded core under a variety of loadings and boundary conditions using a combination of analytical and computational means. Circular sandwich panels with graded core were studied by Sburlati (2012). Exponential variation of Young's modulus through the core thickness was assumed to enable development of analytical solution to the axisymmetric problem. The low velocity impact response of sandwich structures based on layered cores has been studied both experimentally and numerically by Zhou et al. (2013). Thermo-elastic response and free vibrations of cylindrical sandwich panels with graded core were examined by Alibeigloo (2014) and Alibeigloo and Liew (2014). The dynamic responses and blast resistance of all-metallic sandwich plates with functionally graded close-celled aluminium foam cores were investigated using a finite element method and experimentally by Liu et al. (2014). Sandwich panels with lattice core graded in length direction were experimentally studied by Xu et al. (2015).

In the majority of analytical studies on sandwich panels with graded core either an exponential or power-law variation of stiffness properties through the thickness is assumed in order to enable development of the solution. However, there may be benefits in utilising cores with other variations in stiffness properties and so methods must be sought in order to model them. Extending existing methods and solutions to the case of arbitrary variation of stiffness seems to be a natural step in that direction.

In this paper, the three-dimensional elasticity solution for sandwich panels with exponential variation of the Young's modulus through the thickness (Kashtalyan and Menshykova, 2009) is extended to produce a piecewise exponential model that can be applied to any functionally graded plate or sandwich panel with graded core as long as the through thickness stiffness variation is described by a smooth function. Previously, approximation of arbitrary variation of material properties by a piecewise exponential model has been successfully applied to analysis of cracks in functionally graded materials (Guo and Noda, 2007; Bai et al., 2013), thermoelastic deformation of multilayered strips (Ootao, 2011) and plates (Ootao and Ishihara, 2013) and contact mechanics of graded coatings (Liu and Xing, 2014). To the best of our knowledge, this approach has not been applied yet to sandwich panels with graded core in the context of three-dimensional elasticity theory.

A piecewise-exponential model for sandwich panels with graded core presented in this paper is fully validated through both comparison with results from the literature and a finite element study. As an example the new model is applied to analysis of sandwich panels with power law variation in stiffness properties of the core and a study is carried out to examine the effect of varying the power law index on panel's response.

## 2. A piecewise-exponential model

A sandwich panel (Fig. 1) of length $a$, width $b$ and total thickness $h_0 = 2h$ is referred to a Cartesian co-ordinate system $x_1,\ x_2,\ x_3 (0 \leqslant x_1 \leqslant a, 0 \leqslant x_2 \leqslant b, -h \leqslant x_3 \leqslant h)$ and assumed to be symmetric with respect to the mid-plane $x_3 = 0$, with the face sheet thickness $h_f$ and the core thickness $2h_c$. The core of the panel is a functionally graded material, with the shear modulus that varies arbitrarily through the thickness, so that the actual shear modulus at any point is given by

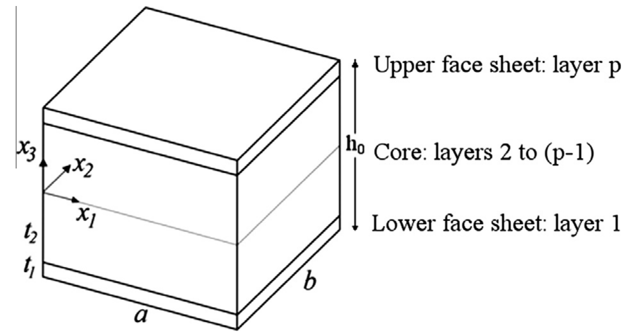


**Fig. 1.** Panel geometry.

$$G(x_3) = \Gamma(x_3) \tag{1}$$

where $\Gamma(x_3)$ is any smooth function. Poisson's ratios of the face sheets and the core are assumed to constant.

Referring to the idea of Guo and Noda (2007), let us divide the panel into $p$ layers $(k = 1, 2 \ldots, p)$, with layers 1 & $p$ being the face sheets and the core subdivided into $p$-2 layers. We assume that within each of the layers, actual variation of the shear modulus can be approximated by an appropriately chosen exponential function. Provided that a sufficiently high number of layers are used and that the actual material properties vary smoothly, then the continuously varying properties can be accurately simulated by the piecewise exponential model.

Let the shear modulus within each layer can be approximated by an exponential function

$$G^{(k)}(x_3) = g^{(k)} \exp\left\{\gamma^{(k)}\left(\frac{x_3}{h} - 1\right)\right\} \tag{2}$$

where $\gamma^{(k)}$ are the inhomogeneity parameters

$$\gamma^{(k)} = \frac{h}{t^{(k)}} \ln \delta^{(k)} \tag{3a}$$

and

$$\delta^{(k)} = \frac{G^{(k+1)}}{G^{(k)}}, \quad g^{(k)} = G^{(k)} \exp\left\{\gamma^{(k)}\left(1 - \frac{h^{(k)}}{h}\right)\right\} \tag{3b}$$

where $t^{(k)}$ is the thickness of the $k$th layer; $G^{(k)}$ and $G^{(k+1)}$ are the shear modulus values at the bottom of the $k$th and $(k+1)$th layers respectively; and $h^{(k)}$ is the height of the bottom of $k$th layer relative to the mid-plane. For ease of modelling, the homogenous face sheets $(k = 1,\ k = p)$ can also be treated as FGMs with the inhomogeneity parameters, $\gamma^{(k)}$, set sufficiently close to zero.

Using the actual material properties defined at each interface given by Eq. (1), expressions (3b) can be rewritten as follows

$$\delta^{(k)} = \frac{\Gamma(h^{(k+1)})}{\Gamma(h^{(k)})}, \quad g^{(k)} = \Gamma(h^{(k)}) \exp\left\{\gamma^{(k)}\left(1 - \frac{h^{(k)}}{h}\right)\right\} \tag{4}$$

Through the coefficients of the exponential functions, the number and thickness of the layers, many different variations in through thickness modulus can be modelled such as power law variation

$$\Gamma(x_3) = G_{tb}\left(\frac{|x_3|}{h_c}\right)^r + G_{core} \tag{5}$$