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The modelling of nonlinear rheological behaviour and Mullins effect in High Damping Rubber

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ABSTRACT

High Damping Rubber (HDR) is used in High Damping Rubber Bearings (HDRB) which, in themselves, are dissipating devices in structural systems. The behaviour of HDRB is essentially linked to that of HDR which combines many phenomena such as nonlinearity, time dependence and memory of the loading history, the so called Mullins effect. Mastering the behaviour of bearings implies an accurate understanding of the response of HDR. Consequently, the purpose of this paper is to present a phenomenological material model for a nonlinear rheological mechanical behaviour accounting for Mullins effect of HDR. Many details of its numerical implementation are given. Based on experimental results and literature review, the Distribution of NonLinear Relaxation (DNLR) approach is explored to develop a new numerical model able to simulate nonlinear rheological response of HDR. Modifying the relaxation time regarding von Mises stress is one of the originalities of this model. A relatively simple way of memorising the loading history is implemented in the finite element program Cast3M© to describe the Mullins effect. Experimental tests are simulated. In order to determine model parameters, strategies for parameter identification are proposed. Firstly, parameters of the nonlinear rheological mechanism are optimised by means of relaxation test simulations and monotonic compression tests. Secondly, parameters of the Mullins effect are identified by using simulation of cyclic tensile tests. Finally, comparisons between experimental and numerical results in quasi-static compression and the cyclic shear of HDR are given in order to validate the model.

1. Introduction

This paper presents a phenomenological material model for High Damping Rubber (HDR) accounting for the Mullins effect. HDR is known to present a nonlinear rheological mechanical behaviour, time dependence and a memory of the loading history at least for a short time. Many details of the numerical implementation are given below. Specifically, the method of modifying relaxation times with von Mises stress and the method of memorising, relatively simply, the loading history in many directions. The formulation is shown to be suitable for simulations of HDR in relaxation tests, cyclic tensile tests, compression tests and under combined quasi-static compression and cyclic shear (QC–CS) under isothermal conditions. These tests are presented in the PhD report of Nguyen (Nguyen, 2013) and reported in the previous paper by Tinard et al. (2015).

HDR is a type of carbon black filled vulcanised natural rubber, commonly used in High Damping Rubber Bearings (HDRB) for base isolation devices which are made of alternating thin horizontal layers of HDR bonded to steel plates (Roeder and Stanton, 1983). Literature review indicates that the behaviour of HDRB is essentially linked to the behaviour of HDR which combines several phenomena such as the non-linearity of stress–strain response (James and Green, 1975; Rivlin and Saunders, 1950; Treolar, 1944), sensitivity to strain rate (Cotton and Boonstra, 1965; Ferry, 1980; Lion, 1996; Miehe and Keck, 2000) and Mullins effect (Amin et al., 2006; Bouasse and Carrière, 1903; Mullins, 1948; Mullins and Tobin, 1965). Due to the complexity of behaviour of HDR, most of the classical implemented mechanical models simulate individual phenomena such as hyperelasticity, elasto-visco-plasticity and Mullins effect. The present model accounts for both elasto-visco-plasticity and Mullins effect but, without loss of ability, it is still restricted to small strains formalism. The goal of this paper

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Abbreviations: HDR, High Damping Rubber; HDRB, High Damping Rubber Bearings; QC–CS, quasi-static compression and cyclic shear; DNLR, Distribution of NonLinear Relaxation; ICQ8, eight node quadratic element.

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is to set up a constitutive model for the description of HDR materials which emphasizes (a) the rheological mechanical behaviour and (b) Mullins effect.

(a) The mechanical behaviour presents two major aspects. The first is the time dependence which controls the relaxation times and leads to strain rate influence. This is also called viscosity (Amin et al., 2006; Rigbi, 1980). It occurs in creep and relaxation tests as well as in dynamic mechanical analysis. The second major contribution to mechanical behaviour is independent of the strain rate and linked to plasticity (Haupt and Sedlan, 2001; Kilian et al., 1994). It is, for instance, observed by a permanent strain after loading. In general, rheological models consist of basic elements such as a spring, dashpot and friction element. These models are known to heavily reproduce the nonlinear mechanical behaviour of rubber. Thus, based on thermodynamic theory, a number of nonlinear rheological models on rubber materials were developed (Amin et al., 2002; Haupt and Sedlan, 2001; Martinez et al., 2011; Miehe and Keck, 2000). However, it has been noticed that rubber recovers its initial state and behaviour with time i.e. the material forgets the loading history with time. Hence, nobody can say that this is true plasticity. The herein proposed model gives an original method of treating "short time" plasticity with a relaxation formalism, combining Eyring's formalism with the use of relaxation times.

(b) Mullins effect was observed for the first time on vulcanized rubber by Bouasse and Carrière (1903). It was then intensively investigated by Mullins (Mullins, 1948, 1969) and is consequently referred to as "Mullins effect". Considering for instance cyclic tests, this phenomenon is characterised by an important loss of stiffness during the first cycles. More precisely:

- The mechanical response is different between cycles, more specifically, between the first and the second. This response is almost stable after a few cycles,
- A stress softening appears each time the maximum strain is increased,
- The material remembers the direction and sign of the maximum strain, at least for a short time,
- The material is able to forget it i.e. to recover almost all of its initial stiffness in time, typically, after several days for our own material.

Several authors developed theories in order to propose physical explanations of this effect, including chain scission (Bueche, 1960), molecule slipping (Houwink, 1956) and chain disentanglements (Hamed and Hatfield, 1989). In general, the description of the Mullins effect is based on two main approaches (Cantournet et al., 2009; Qi and Boyce, 2004). The first approach was initially proposed by Mullins and Tobin (1957, 1965): the material is assumed to be constituted of a rigid phase and a soft phase. Under loading, part of the rigid phase is transformed into the soft phase due to the rearrangement of molecular networks. Harwood et al. (1965), Harwood and Payne (1968) introduced a law of evolution of volume fraction of the soft phase by the maximum strain applied. Miehe (1995), Qi and Boyce (2004) generalised this approach and implemented this law of evolution in finite element models. This approach is often used in microscopic models. The second approach focuses on damages to materials by way of breaks between the polymer chains or by the destruction of bonds between the polymer chains and the filler particles (Govindjee and Simo, 1991; Miehe and Keck, 2000). Mullins effect can, hence, be interpreted as a damage effect. The variation of damage is often described by a function of maximum stretch or maximum stress or invariant obtained in the loading history (Andrieux et al., 1997; Machado et al., 2010; Ogden and Roxburgh, 1999).

In order to reach this study's objective, constitutive equations of an original model are firstly presented. Secondly, geometry and boundary conditions to simulate the test such as relaxation tests, compression tests and QC–CS tests are introduced. Based on experimental results, strategies for parameter identification of the model are suggested. Some experimental results (relaxation tests, compression tests and cyclic tensile tests) are used to optimise the parameters of the model and other experimental results (QC–CS tests) are used to validate the model.

2. Constitutive model description

Suggested by the experimental results (Nguyen, 2013; Tinard et al., 2015) and the literature review of rubbers mechanical modelling, the Distribution of NonLinear Relaxation (DNLR) approach proposed by Cunat (1988) is used to develop a numerical model for simulating nonlinear rheological mechanical behaviour in HDR. Then, the model is enriched to describe the Mullins effect. Finally, it is implemented in the finite element program Cast3M© to simulate some experimental tests.

2.1. Nonlinear rheological model

As previously mentioned, there are many numerical models for simulating the nonlinear rheological behaviour of materials. Furthermore, the previously performed QC–CS tests (Tinard et al., 2015) indicate that compression stress decreases with oscillation during the cyclic shear test, as illustrated in Fig. 1. Compression of the sample induces shearing between the steel plates near the borders and hydrostatic stress in the centre. When the compressed sample is submitted to cyclic shear, the raise in shear stress activates relaxation processes. This leads to macromolecular reorganisation which tends to stabilise in a relaxed state but this relaxed state is continuously modified during the test. It may explain the observed oscillations of compression stresses during cyclic shear. This is a material behaviour effect enhanced by the structure of HDRB.

Therefore, this phenomenon led to the concept of activation energy following Eyring formalism (Halsey et al., 1945) and to combine it with the use of the DNLR approach developed by Cunat (Cunat, 2001; Lemaitre, 2001; Mrabet et al., 2005). Within the framework of thermodynamics of irreversible processes, this approach is based on the generalisation of Gibbs' relation for out – of – equilibrium situations (Münster, 1966). Details on the development of constitutive laws are given in Cunat (1996, 2001). An illustration of Cunat's model is presented in Fig. 2. In this model, relaxation times are modified by stress state. This means that the relaxed state of the HDR is modified during cyclic shear. Hence, by using such activation, long time relaxation takes the place of classical plasticity. Moreover, it permits the taking into account of the recovering of Mullins effect over time.

The isothermal model developed by Cunat is written in three-dimensional form and is transformed into the following equation:

$$\dot{\sigma}_{ij(q)} = p_{(q)} C^{u}_{ijkl}\dot{\varepsilon}_{kl} - (\sigma_{ij(q)} - p_{(q)}\sigma^{r}_{ij(q)})/\tau_{(q)} \tag{1}$$

where

- $\dot{\sigma}_{ij(q)}$ is the stress rate response of mode q,
- $\sigma_{ij(q)}$ is the total in the q-branch actual stress response,
- $\sigma^{r}_{ij(q)}$ is the relaxed stress,
- $\dot{\varepsilon}_{kl}$ is the strain rate,
- $\tau_{(q)}$ is the relaxation time of mode q,
- $p_{(q)}$ is a weighted coefficient to quantify the contribution of each mode in the overall stress response. $p_{(q)}$ is chosen in such a way that $\sum_{j=1}^{N} p_{(q)} = 1$ where N is the number of modes,
- C^{u}_{ijkl} represents the instantaneous stiffness tensor.