International Journal of Solids and Structures 75–76 (2015) 299–308

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Wave propagation across the imperfectly bonded interface between cracked elastic solid and porous solid saturated with two immiscible viscous fluids

Sushant Shekhar *, Imtiyaz A. Parvez

Academy of Scientific and Innovative Research, New Delhi 110 001, India
CSIR-Fourth Paradigm Institute, Bangalore 560037, India

ARTICLE INFO

Article history:
Received 10 January 2015
Received in revised form 8 July 2015
Available online 5 September 2015

Keywords:
Reflection and refraction
Energy partition
Seismic attenuation
Cracked elastic solid
Saturated porous solid

ABSTRACT

The present study is aimed at understanding the effect of a vertically aligned crack, present in the elastic half space on the propagation of attenuated waves. These waves are incident at a point on the interface between the porous half space and the cracked elastic half space. The analysis is based on Snell's law for reflection and refraction of an incident wave at the interface. A loose bonding at the interface between the porous half space and the cracked elastic half space has been considered and represented here as the tangential slip. The proposed model is solved for the propagation of harmonic plane waves. The final equations are in the form of Christoffel equations from which we find four reflected waves (three longitudinal body waves and one transverse body wave) and two refracted waves (one longitudinal body wave and one transverse body wave). The expression of reflection–refraction coefficients and energy share of each reflected and refracted waves for a given incident wave is obtained in closed form and computed numerically in the present study. Numerical examples are considered for the partition of the incident energy in which we have studied the effect of aspect ratio, crack density and loose bonding parameter.

1. Introduction

Poroelasticity theory is an important theory for the study of the mechanical behavior of porous solids in different fields, some of which are soil dynamics, oil exploration, earthquake engineering, geomechanics and reservoir engineering. The study of wave propagation in a porous solid saturated with a single fluid was started by Biot who has published some important research work related to wave propagation in porous media, Biot (1956a,b, 1962a,b) and Biot and Willis (1957). From his studies Biot has found two longitudinal body waves and one transverse body wave.

Biot's theory has been extended as mixture theory in which the porous medium is saturated by more than one fluid. Brutsaert (1964) first found the presence of a third longitudinal body wave in an unsaturated granular medium. Based on this study, Bedford and Drumheller (1983) have developed theories of immiscible and structured mixtures. Garg and Nayfeh (1986) have discussed the third compressional wave in their study. For low frequency elastic waves, Tuncay and Corapcioglu (1997) have successfully studied the presence of three longitudinal body waves and one transverse body wave in a porous solid saturated with two immiscible fluids for which they had used a volume averaging technique. They found that the first two longitudinal body waves are the same as Biot (1956b) while the third longitudinal body wave is due to the presence of the third fluid. Using this theory, many developments have been carried out by researchers; e.g. Yew and Jogi (1976), Tomar and Arora (2006) and Sharma and Saini (2012).

Deresiewicz (1962) has studied the effect of the boundaries of the liquid filled porous solid on the propagation of a wave that changes the wave pattern of the elastic wave. Deresiewicz and Skalak (1963) have successfully applied Neumann's uniqueness theorem of elasticity to a porous medium for defining the boundary conditions. Based on the previous study, Sharma (2009) has given different cases for the boundary conditions for the porous solid. A study of the reflection and transmission from the interface between two media have been carried out by some researchers e.g. Ainslie and Burns (1995), Borcherdt (1982), Berryman (2007), Denneman et al. (2002), Sharma and Gogna (1991), Sharma (2008a) and Vashisth et al. (1991).

The earth's crust normally has a lot of aligned cracks or micro-cracks which contain fluids or sometimes voids. O'Connell and Budiansky (1974) calculated the effect of cracks on the elastic

* Corresponding author at: CSIR-Fourth Paradigm Institute, Bangalore 560037, India. Tel.: +91 8025051346; fax: +91 8025220392.
E-mail address: sushantshekhar85@yahoo.com (S. Shekhar).

http://dx.doi.org/10.1016/j.ijsolstr.2015.08.022
0020-7683/

properties of an isotropic solid. Most of the cracks are developed due to small earthquakes that have been described in Crampin (1978, 1985, 1987) who found some important aspects related to wave propagation through these cracks. These small earthquakes may result due to the accumulation of stresses in the particular region. The study of cracks inside the earth's crust can give important information related to the oils/water/minerals deposited in these cracks. Fluid flow from a porous medium to the cracked elastic medium can be controlled by some boundary conditions at the interface, those may be fully opened or closed or partially open-closed, as given in Sharma and Gogna (1991), Sharma (1999, 2008a, 2009) and Vashisth et al. (1991). The presence of the attenuation plays an important role in wave propagation through aligned cracks and this phenomena has been shown in Chatterjee et al. (1980) and Xu and King (1990). Currently, Nandan and Saini (2012) have studied the effect of an aligned crack on the reflection and transmission of elastic waves through the interface between the poroelastic solid with one fluid and cracked elastic solid.

For the study of wave propagation, we summarize the previous work in Table 1 based on appropriate criteria. In the present study, we consider an isotropic homogeneous poroelastic medium saturated with a mixture of two immiscible fluids lying over the cracked elastic half space. We assume that the two media are loosely connected to each other and the connected coefficient or bonding parameter is represented here by ψ. The interface between these two media is assumed at $x_3 = 0$. We solve the dynamical equation with the help of the assumed harmonic solution. The obtained results are in the form of Christoffel equations and these results provide four inhomogeneous waves in a porous medium, of which three are longitudinal body waves and one is a transverse body wave. The reflection coefficients and energy share have been solved for given boundary conditions at a loosely bonded interface. The energy matrix defines the distribution of the incident energy to the four reflected waves, two refracted waves and some energy is spent at the interface and is defined as dissipation energy. The final results related to energy share satisfy the conservation law of energy. We graphically demonstrate the results of energy share with respect to the incident angle θ for the effect of aspect ratio c/a (where c is the crack thickness and a is the radius of circular crack), crack density η and bonding parameter ψ. We have also conducted a comparative study between the presence and absence of vertical aligned cracks with respect to the crack density and the crack thickness in the elastic half space. For the numerical validation of the present study, we assume that the first medium is water and CO_2 saturated sandstone and second medium is basaltic rock.

Table 1
Classification of related references by type of systems and chronological order.

Solid	**Porous media**
• Biot and Willis (1957) • Achenbach (1973) • O'Connell and Budiansky (1974) • Crampin (1978, 1985, 1987) • Xu and King (1990) • Ainslie and Burns (1995) • Sharma (1999)	• Biot (1956a,b, 1962b,a) • Vashisth et al. (1991) • Tuncay and Corapcioglu (1997) • Denneman et al. (2002) • Tomar and Arora (2006) • Nandan and Saini (2012) • Sharma and Saini (2012)
Ideal fluids	**Viscous fluids**
• Sharma (2008a) • Nandan and Saini (2012)	• Chatterjee et al. (1980) • Tomar and Arora (2006) • Sharma and Saini (2012)
Perfect interface	**Imperfect interface**
• Ainslie and Burns (1995) • Denneman et al. (2002) • Tomar and Arora (2006) • Sharma and Saini (2012)	• Vashisth et al. (1991) • Nandan and Saini (2012)
Isotropic	**Anisotropic**
• Tuncay and Corapcioglu (1997) • Tomar and Arora (2006) • Nandan and Saini (2012) • Sharma and Saini (2012)	• Biot and Willis (1957) • Sharma and Gogna (1991) • Vashisth et al. (1991) • Sharma (2008b)

2. Basic theoretical framework

2.1. Poroelastic solid with two immiscible fluids

The balance equation in the absence of body forces for the low frequency vibration in a tri phase solid–air–water mixture can be expressed as Tuncay and Corapcioglu (1997),

$$\begin{aligned}
\langle\tau_s\rangle_{ij,j} &= \langle\rho_s\rangle\frac{\partial^2 u_i}{\partial t^2} - d_g\frac{\partial}{\partial t}(v_i - u_i) - d_l\frac{\partial}{\partial t}(w_i - u_i)\\
\langle\tau_g\rangle_{ij,j} &= \langle\rho_g\rangle\frac{\partial^2 v_i}{\partial t^2} + d_g\frac{\partial}{\partial t}(v_i - u_i)\\
\langle\tau_l\rangle_{ij,j} &= \langle\rho_l\rangle\frac{\partial^2 w_i}{\partial t^2} + d_l\frac{\partial}{\partial t}(w_i - u_i)
\end{aligned}\tag{1}$$

where the subscripts s, g, l define solid, gas and liquid phases, respectively. For phase $k(= s, g, l)$, $\langle\tau_k\rangle$'s and $\langle\rho_k\rangle$'s signify the average stresses and average partial density over the solid–gas–liquid aggregate. u_i, v_i and w_i represent the displacement components of solid, gas and liquid particles. Here, the coefficients d_g and d_l define the presence of dissipation related to gas and liquid particles in the porous medium (according to Darcy's law) and these coefficients can be defined here as:

$$d_k = \frac{\mu_k\alpha_k^2}{\vartheta\vartheta_k}, \quad (k = g, l)\tag{2}$$

where the symbol's μ_k, ϑ_k, and α_k represent the viscosity, relative permeability and volume fractions for each fluid and ϑ represents the intrinsic permeability of the porous medium. The stresses in the porous solid and the fluid pressures in the pores can be given as:

$$\begin{aligned}
\langle\tau_s\rangle_{ij} &= (a_{11}u_{k,k} + a_{12}v_{k,k} + a_{13}w_{k,k})\delta_{ij} + a_{10}(u_{i,j} + u_{j,i})\\
\langle\tau_g\rangle_{ij} &= (a_{21}u_{k,k} + a_{22}v_{k,k} + a_{23}w_{k,k})\delta_{ij}\\
\langle\tau_l\rangle_{ij} &= (a_{31}u_{k,k} + a_{32}v_{k,k} + a_{33}w_{k,k})\delta_{ij}
\end{aligned}\tag{3}$$

where δ_{ij} is the Kronecker symbol. a_{10} and $a_{ij}(i, j = x, y, z)$ are said to be elasticity constants with property $a_{ij} = a_{ji}$, and can be written as:

$$\begin{aligned}
&a_{10} = G_{fr}, \quad a_{11} = K_{fr} - \frac{2}{3}G_{fr}, \quad a_{12} = a_{21} = K_g S_g \alpha_s (K_l + \gamma)/K,\\
&a_{13} = a_{31} = K_l\alpha_s(1 - S_g)(K_g + \gamma)/K, \quad a_{22} = K_g\alpha_g(K_l S_g + \gamma)/K,\\
&a_{23} = a_{32} = K_g K_l S_g \alpha_l/K, \quad a_{33} = K_l\alpha_l[K_g(1 - S_g) + \gamma]/K\\
&K = K_g(1 - S_g) + K_l S_g + \gamma, \quad \gamma = (1 - S_g)K_{cap}
\end{aligned}$$

where K_{cap} is called the macroscopic capillary pressure, Garg and Nayfeh (1986). K_{fr}, K_g and K_l are said to be the bulk modulus of the porous frame, gas phase and liquid phase, respectively. G_{fr} denotes the shear modulus for the porous solid. $S_i = \alpha_i/(1 - \alpha_s)(i = g, l)$ with $S_g + S_l = 1$ are the gas saturation and liquid saturation for the porous solid.

In terms of the displacement components, the equations of motion are expressed using Eq. (3) in Eq. (1):

$$\begin{aligned}
&(a_{10} + a_{11})u_{j,ij} + a_{12}v_{j,ij} + a_{13}w_{j,ij} + a_{10}u_{i,jj}\\
&\quad = \langle\rho_s\rangle\frac{\partial^2 u_i}{\partial t^2} - d_g\frac{\partial}{\partial t}(v_i - u_i) - d_l\frac{\partial}{\partial t}(w_i - u_i)\\
&a_{21}u_{j,ij} + a_{22}v_{j,ij} + a_{23}w_{j,ij} = \langle\rho_g\rangle\frac{\partial^2 v_i}{\partial t^2} + d_g\frac{\partial}{\partial t}(v_i - u_i)\\
&a_{31}u_{j,ij} + a_{32}v_{j,ij} + a_{33}w_{j,ij} = \langle\rho_l\rangle\frac{\partial^2 w_i}{\partial t^2} + d_l\frac{\partial}{\partial t}(w_i - u_i)
\end{aligned}\tag{4}$$

For solving Eq. (4) harmonically, we assume the displacements component as