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Peridynamics for bending of beams and plates with transverse shear deformation

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ABSTRACT

Progressive failure analysis of structures is still a major challenge. There exist various predictive techniques to tackle this challenge by using both classical (local) and nonlocal theories. Peridynamic (PD) theory (nonlocal) is very suitable for this challenge, but computationally costly with respect to the finite element method. When analyzing complex structures, it is necessary to utilize structural idealizations to make the computations feasible. Therefore, this study presents the PD equations of motions for structural idealizations as beams and plates while accounting for transverse shear deformation. Also, their PD dispersion relations are presented and compared with those of classical theory.

1. Introduction

Peridynamic (PD) theory was originally introduced for the solution of deformation field equations (Silling, 2000) without any structural idealizations. It satisfies all the fundamental balance laws of classical (local) continuum mechanics; however, it is different in the sense that it is a nonlocal continuum theory and it introduces an internal length parameter into the field equations. This internal length parameter defines the association among the material points within a finite distance through micropotentials. Removal of micropotentials between the material points allows damage initiation and growth through a single critical failure parameter regardless of the mixed-mode loading conditions. The creation of a new (crack) surface is based on a local damage measure. The local damage is defined as the ratio of broken interactions to the total number of interactions at a material point.

Finite element analysis (FEA) with traditional elements suffers from the following shortcomings: (1) the interface between dissimilar materials is assumed to have zero thickness without any specific material properties; however, it presents a weak link and it is usually the location of failure. Therefore, it fails to appropriately model the interface between dissimilar materials. (2) Failure is a dynamic process, and it requires remeshing. It is computationally costly, and the crack growth is guided based on the linear elastic fracture mechanics (LEFM) concepts. It breaks down when multiple complex crack growth patterns develop. (3) Stress and strain fields are discontinuous, and mesh refinement does not necessarily ensure accurate stress fields near geometric and material discontinuities. (4) Finally, crack nucleation is not resolved. The analysis always requires a pre-existing crack.

In order to remedy or remove these shortcomings, Cohesive Zone Elements (CZE) and eXtended Finite Elements (XFEM) were developed; however, CZE requires a priori knowledge of the crack path. In a complex analysis, it is not practical and the results are dependent on the mesh (structured or unstructured). Furthermore, the results are sensitive to the strength parameters in the traction–separation law of the cohesive zone model. Determination of these parameters poses additional uncertainties. Although XFEM removed such uncertainties, it still requires an external criteria for crack propagation. Thus, the results depend on the criteria employed in the analysis. It also breaks down when multiple complex crack growth patterns develop.

The PD theory overcomes the weaknesses of the existing methods, and it is capable of identifying all of the failure modes without simplifying assumptions. The PD methodology effectively predicts complex failure in complex structures under general loading conditions. Damage is inherently calculated in a PD analysis without special procedures, making progressive failure analysis more practical.

An extensive literature survey on PD is given in a recently published textbook by Madenci and Oterkus (2014). A comparison study between peridynamics, CZE, and XFEM techniques is given by Agwai et al. (2011). They showed that the crack speeds obtained from all three approaches are on the same order; however, the

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fracture paths obtained by using peridynamics are closer to experimental results with respect to other two techniques.

Another advantage of PD is its length-scale parameter, which does not exist in classical continuum mechanics. Such a length-scale parameter gives PD a nonlocal character. Hence, it allows the capture of physical phenomena not only at the macro-scale, but also at various other scales. This characteristic can be established through the PD dispersion relations. The classical theory is only valid for a special case of a long wavelength limit; however, the PD shows dispersion behavior similar to that observed in real materials. Hence, it is proven to be acceptable to perform multi-scale analysis simulations.

Although peridynamics is a powerful technique in failure analysis and has an internal length scale, it is usually computationally more expensive, especially with respect to finite element analysis. The computational time can be significantly reduced by using parallel computing either by using a CPU (Central Processing Unit) and/or GPU – based (Graphics Processing Unit) architecture for which PD equations of motion are very suitable. However, modeling very large and detailed structures such as aerospace and marine vehicles can still be computationally demanding. Hence, in such cases it is necessary to reduce computational time through structural idealization. Taylor and Steigmann (2013) proposed a peridynamic plate model based on bond-based formulation by using an asymptotic analysis. Their formulation is capable of capturing out-of-plane deformations for thin plates. Moreover, O'Grady and Foster (2014a,b) developed a non-ordinary state-based peridynamic model for Euler–Bernoulli beam and Kirchhoff–Love plate formulations by disregarding the transverse shear deformations. Therefore, the focus of this study is to present a new PD formulation for thin or thick beams and plates while accounting for transverse shear deformation based on an original (bond-based) PD formulation. Moreover, PD dispersion relations are obtained and compared against those from classical theory.

The following sections present the PD kinematics for a Timoshenko beam and a Mindlin plate, and the corresponding PD equations of motion as well as the PD material parameters. They also describe the procedure to determine the surface correction factors for these parameters and the application of the boundary conditions and determination of the critical curvature and critical shear angle in terms of the fracture mechanics parameters. Finally, the corresponding dispersion relations are derived and compared with the classical theory. The numerical results establish the validity of the present formulation by considering simple benchmark problems.

2. Peridynamic kinematics

At any instant of time, every point in the beam or plate denotes the out-of-plane deflection and rotations of a material particle, and these infinitely many material points (particles) constitute the beam or the plate. In the undeformed state of the body, each material point is identified by its coordinates, $\mathbf{x}_{(k)}$ with $(k = 1, 2, \ldots, \infty)$, and is associated with an incremental volume, $V_{(k)}$, and a mass density of $\rho(\mathbf{x}_{(k)})$. According to the PD theory introduced by Silling (2000), the motion of a body is analyzed by considering the pair-wise interaction between material points $\mathbf{x}_{(k)}$ and $\mathbf{x}_{(j)}$. The interaction between the material points is prescribed through a micropotential that depends on the deformation and constitutive properties of the material. Also, a material point is only influenced by the other material points within a neighborhood defined by its horizon, δ. The micropotentials are zero for material points outside its horizon. Each material point can be subjected to prescribed body loads, displacement, or velocity, resulting in motion and deformation.

2.1. Beam kinematics

As shown in Fig. 1, the transverse shear angles, $\varphi_{(j)}$ and $\varphi_{(k)}$, of material points j and k can be expressed as

$$\varphi_{(j)} = \left(\frac{w_{(j)} - w_{(k)}}{\xi_{(j)(k)}} - \phi_{(j)}\mathrm{sgn}(x_{(j)} - x_{(k)})\right) \tag{1a}$$

$$\varphi_{(k)} = \left(\frac{w_{(j)} - w_{(k)}}{\xi_{(j)(k)}} - \phi_{(k)}\mathrm{sgn}(x_{(j)} - x_{(k)})\right) \tag{1b}$$

in which $w_{(j)}$, $\phi_{(j)}$ and $w_{(k)}$, $\phi_{(k)}$ represent the out-of-plane deflection and rotation of material points j and k, respectively. The distance between the material points j and k is specified as $\xi_{(j)(k)} = |x_{(j)} - x_{(k)}|$.

Considering the material point k as the point of interest, the transverse shear angle, $\varphi_{(k)(j)}$, arising from the interaction between material points j and k can be defined as the average of the transverse shear angles at these material points in the form

$$\varphi_{(k)(j)} = \left(\frac{w_{(j)} - w_{(k)}}{\xi_{(j)(k)}} - \frac{\phi_{(j)} + \phi_{(k)}}{2}\mathrm{sgn}(x_{(j)} - x_{(k)})\right) \tag{2}$$

The curvature between the material points j and k can be defined as

$$\kappa_{(k)(j)} = \left(\frac{\phi_{(j)} - \phi_{(k)}}{\xi_{(j)(k)}}\right) \tag{3}$$

When considering the material point j as the point of interest, the transverse shear angle and curvature for the interaction between the material points j and k become

$$\varphi_{(j)(k)} = \left(\frac{w_{(k)} - w_{(j)}}{\xi_{(j)(k)}} - \left(-\frac{\phi_{(k)} + \phi_{(j)}}{2}\right)\mathrm{sgn}(x_{(j)} - x_{(k)})\right) \quad \text{or}$$
$$\varphi_{(j)(k)} = -\varphi_{(k)(j)} \tag{4a}$$

and

$$\kappa_{(j)(k)} = \left(\frac{\phi_{(k)} - \phi_{(j)}}{\xi_{(j)(k)}}\right) \quad \text{or} \quad \kappa_{(j)(k)} = -\kappa_{(k)(j)} \tag{4b}$$

2.2. Plate kinematics

As illustrated in Fig. 2, $\phi_{(j)}$ and $\phi_{(k)}$ represent the rotations with respect to the line of action between the material points j and k. Considering the material point k as the point of interest, the curvature, $\kappa_{(k)(j)}$, with respect to the line of action between the material points j and k can be defined as

$$\kappa_{(k)(j)} = \frac{\phi_{(j)} - \phi_{(k)}}{\xi_{(j)(k)}} \tag{5}$$

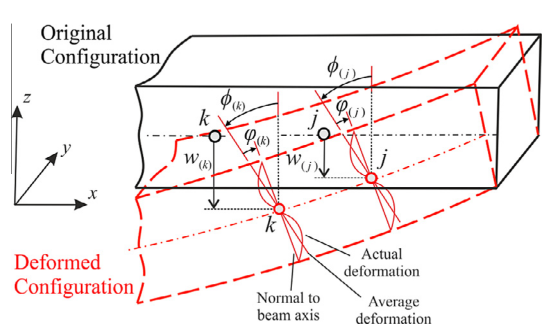

Fig. 1. Original and deformed configurations of a Timoshenko beam.