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Analytical micro-modeling of masonry periodic unit cells – Elastic properties

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ABSTRACT

A model for the analysis of masonry periodic unit cells based on detailed micro-modeling principles is presented: an analytical model capable of calculating the orthotropic elastic properties of masonry composites. Several masonry typologies are investigated in the present paper, some of which have received very limited attention, such as Flemish bond walls.

The purpose of the model is to achieve the level of detail made possible by the use of detailed micro-modeling techniques while keeping computational costs at a minimum in order to be used as a basis for two-scale modeling of masonry.

The results obtained from the described model are compared with experimental data from the existing relevant literature as well as with numerical results obtained from the finite element analysis of masonry periodic unit cells. A parametric investigation is also performed, in which the accuracy of the model is compared against a FE benchmark.

1. Introduction

1.1. State of the art

Masonry structures currently constitute a very large part of the existing built environment. They include a large amount of common but still-in-use buildings as well as outstanding cultural heritage structures. Masonry buildings are comprised of structural members such as load bearing walls, pillars and vaults composed of single or multiple brick or stone masonry leaves with possible interior rubble infills. An accurate structural analysis of such buildings is important as a way to verify their capacity against gravity loads and horizontal actions such as wind and earthquake. However, the structural analysis of masonry buildings faces significant challenges due to the complex behavior of masonry which stems from its composite character, the influence of the geometric arrangement of units and the fact that most utilized mortars only provide a weak bond between units. One of the consequences of the composite nature of masonry is found in its anisotropic mechanical response. Another emerging consequence is the difficulty of predicting the average masonry mechanical properties, such as, in particular, the masonry compressive strength and the Young's modulus, from the properties of the constituent materials. Even when the elastic properties of the constituent materials are known, it is difficult to derive the elastic properties of the composite without resorting to sophisticated analysis tools. Furthermore, results for one masonry typology may not be readily suitable for the evaluation of the properties of another. Therefore, the development of simple tools for the derivation of the orthotropic elastic properties of a variety of masonry typologies based on the elastic properties of their constituent materials, while maintaining a single analysis methodology throughout, constitutes a sound framework for the further development of analysis tools based on the detailed micro-modeling approach.

Masonry structures composed of periodically repeating patterns may be simplified in order to facilitate their analysis. In this sense, it is possible to calculate the elastic and strength properties of masonry structures through the analysis of a geometrically repeating part. This part may be further simplified by taking into account symmetry arising from geometrical and loading conditions.

A number of analytical and numerical models has been proposed for the analysis of masonry periodic unit cells using a variety of methods. These have been employed for the derivation of the elastic and inelastic properties of masonry composites as a result of the interaction of their two phases: units and mortar. The cells are analyzed considering appropriate boundary conditions, kinematic compatibilities and stress equilibrium in order to derive

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Notation

E_u	Young's modulus of units
E_m	Young's modulus of mortar
E_i	Young's modulus of infill
$E_{c,i}$	Young's modulus of masonry in direction i
G_u	shear modulus of units
$G_{c,ij}$	shear modulus of masonry in plane ij
ν_u	Poisson's ratio of units
ν_m	Poisson's ratio of mortar
ν_i	Poisson's ratio of infill
$\nu_{c,ij}$	Poisson's ratio of masonry: contraction in direction j for applied extension in direction i
σ_{ii}	normal stress in direction i
σ_{ij}	shear stress in plane ij
h_u	height of units
l_u	length of units
t_u	width of units
h_m	thickness of mortar bed joint
l_m	thickness of mortar head joint
t_m	thickness of transversal mortar joint
t_i	transversal thickness of infill
D_i	dimension of cell in direction i
d_i	deformation of cell in direction i
d_{pn}	displacement vector of node n
$d_{pn,i}$	displacement of node n in direction i
d_i^{ij}	deformation of cuboid in direction i due to shear strain in plane ij

the strains and stress state in the cell for different loading conditions. Nonlinear properties may be derived from the iterative solution of the problem under the assumptions of plasticity or damage models for the behavior of the materials in the cell.

Through these computations in the micro- or meso-scale it is possible to calculate the orthotropic behavior of the masonry composite in the macro-scale, arising from the geometrical arrangement of the two phases in the cell. Elastic (Anthoine, 1995; Briccoli Bati et al., 1999b; Cecchi and Sab, 2002; Lopez et al., 1999) and strength properties (Sacco, 2009; Milani et al., 2006; Cavalagli et al., 2011; Massart et al., 2005) of the composite for in-plane and out-of-plane load conditions have been calculated using such techniques. Research has extended from the study of periodic masonries to non-periodic masonries (Cavalagli et al., 2011; Cluni and Gusella, 2004).

Methods for the analysis of masonry cells using analytical methods have been proposed, mostly focusing on the analysis of stack bond and running bond masonry wall typologies (Lopez et al., 1999; Pande et al., 1989; Briccoli Bati et al., 1999a). Finite element representations of masonry unit cells have also been proposed (Massart et al., 2004) in which the interaction of the two phases, the resulting stress and strains in the cell and the nonlinear behavior of the materials may be accurately represented. This approach has also been adopted for the verification of the accuracy of analytical models, such as the ones already described, the production of in-plane strength domain curves for masonry membranes and the execution of two-scale analyses. However, FE calculations may require time for the creation of the models and high computational effort. A comparison of analytical model and FE model results may be found in Cecchi and Sab (2002).

A micromechanical approach for the analysis of periodically reinforced composites, according to which a repeating cell of the composite is discretized into sub-cells with different properties and arranged in a regular grid, has been proposed in the past (Aboudi, 1991). Equilibrium and compatibility conditions are assembled in a set of closed form expressions and can be solved in a single analysis step for the derivation of, for example, the average elastic properties of the composite. The benefit of this approach is its computational efficiency and relative simplicity. Masonry periodic unit cells, seen as regular arrangements of square or cubic sub-cells with different material properties, can be analyzed using the same approach. Adaptations of this method, capable of providing closed form expressions for the elastic properties of stack and running bond masonry, have been proposed (Taliercio, 2014; Zucchini and Lourenço, 2002).

Masonry analyzed in this manner is usually idealized as an infinitely thin or infinitely thick membrane, these assumptions being accordingly equivalent to a plane stress and plane strain approach. However, the existence of transversal joints, gaps or other discontinuities, which result in non-constant geometric structure along the depth of the masonry, render these two approaches fundamentally not accurate. Analysis of the unit cell taking into account these discontinuities must consider the actual finite thickness and actual geometry of the masonry structure.

Computations on cells where the actual finite thickness of the masonry is considered allow for a more accurate representation of out-of-plane stresses, which, while only marginally affecting the initial elastic stiffness, may strongly influence the compressive strength of the composite (Addessi and Sacco, 2014). However, since it is intended to apply the models proposed here in nonlinear analysis in a following paper, it is deemed necessary to include a realistic representation of the out-off-plane stresses using three-dimensional models for linear elastic analysis as well. With this type of models, the true thickness of the masonry may be easily taken into account in finite element calculations. Through computations on periodic unit cells it is possible to make fairly accurate predictions of the compressive and tensile strength and the elastic moduli of masonry. Such techniques may be, therefore, used for two-scale modeling of masonry walls (Zucchini and Lourenço, 2009; Marfia and Sacco, 2012; Massart et al., 2007; Milani et al., 2006; Addessi and Sacco, 2012).

1.2. Objectives

The purpose of this paper is to present a simple analytical calculation method for the derivation of the elastic characteristics of masonry structures. The method is based on the analysis of masonry periodic unit cells, the smallest repeating geometrical entity representative of the overall masonry geometric pattern, in an attempt to derive masonry composite orthotropic macro-properties from material and geometrical micro-properties. The micro properties considered are those of the units, the mortar and the infill. Structural members allowing this type of modeling include single- and multi-leaf walls and pillar-like structures. These properties are to be determined for normal and shear loading. The models are formulated based on three-dimensional elasticity in order to include the influence of out-of-plane stresses on the response.

These macro-properties may be used for the analysis of large walls and other structures in full multi-scale models or may be used to provide the information needed for analysis with orthotropic material models, such as the Hill, Rankine–Hill or the Hoffmann criteria. The computational cost associated with FE analysis of cells in two-scale analyses is still relatively high, especially when the