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An efficient solution procedure for crushing failure in 3D limit analysis of masonry block structures with non-associative frictional joints

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ABSTRACT

A formulation for limit analysis of three-dimensional masonry structures modelled as rigid block assemblages is presented. A concave contact model is adopted for interfaces, using contact points located at the corners of the interface to represent interactions. A no-tension and non-associative frictional behaviour with limited compressive strength is considered for joints. The limit analysis problem is formulated as a second order cone programming problem (SOCP) and an iterative procedure is proposed to model crushing failure and to take into account non-associative frictional behaviour. Applications to numerical case studies are presented for validation. Finally, the accuracy and the computational efficiency of the proposed formulation are evaluated by a comparison with the results of a full scale experimental sub-assemblage of a masonry pier-spandrel system.

1. Introduction

Rigid block limit analysis is an efficient computational strategy to assess the ultimate loads and failure mechanisms at incipient collapse of masonry structures.

When a micro-modelling approach is adopted, masonry units are modelled with rigid blocks interacting at contact interfaces that, in case of masonry structures with dry or poor quality mortar joints, can be treated using a no-tension and frictional behaviour with infinite or limited compressive strength.

The calculation of the collapse loads and failure modes can be cast as an optimization problem, formulated on the basis of the kinematic and static conditions governing the rigid block model, which can be efficiently solved using mathematical programming methods (Livesley, 1978, 1992; Gilbert and Melbourne, 1994; Baggio and Trovalusci, 2000; Ferris and Tin-Loi, 2001; Orduña and Lourenço, 2005a; Whiting et al., 2009).

The use of mathematical programming for rigid block analysis of masonry structures is receiving a new impulse in recent years.

This is mainly due to the availability of efficient algorithms based on interior point methods for the solution of linear as well as non-linear mathematical programming problems (Andersen et al., 1996, 2003). Besides, the development of simplified formulations and iterative procedures to tackle with the different source of non-linearities, allows to further speed up solution times, as illustrated in Gilbert et al. (2006), Casapulla et al. (2013) and Portioli et al. (2013a,b) for modelling the non-associative behaviour in sliding, the interaction effects of contact forces or even the failure of masonry units.

The mathematical problem that is associated to calculation of the collapse load and failure mechanism in rigid block analysis is the so-called mixed complementarity problem (MCP). This is defined by collecting statics, kinematics and constitutive relationships that govern the behaviour of the rigid block assemblage. The constitutive laws governing the behaviour at contact interfaces can be defined using limit conditions for stress states and suitable relations for strain rates, analogous to yield functions and flow rules in classical plasticity.

When a non-associative behaviour is assumed for displacement rates, as for frictional contacts with a non-dilatant behaviour (non-associative friction), a lower and hence safe value of the collapse load multiplier is generally obtained, if compared to the associative solution. In such a case the solution of MCP is not unique and the problem of minimizing the collapse load arises. The optimization problem resulting from the minimization of the collapse load represents a special class of mathematical programming problems known as mixed complementarity problem with equilibrium constraints (MPEC).

The solution of the MPEC problem is a difficult task and generally involves long CPU times when assemblages with a large number of rigid blocks are considered.

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Moreover, it has been pointed out that solution procedures of the MPEC problem based on the minimization of the load factor (such as that proposed in Ferris and Tin-Loi, 2001) might result in ultimate load factors which can remarkably underestimate the actual behaviour in the case of 3D problems (Orduña and Lourenço, 2005b).

To overcome difficulties related to the solution of the MPEC problem and to save CPU time, a novel formulation has been also proposed for the limit analysis of large three-dimensional masonry block assemblages by Portioli et al. (2014). The model is based on a 'point-contact' formulation, also known as 'concave contact' formulation according to the definition given by Livesley (1992), using contact points located at the corners of the interface to represent interactions. A tensionless, frictional behaviour with infinite compressive strength is assumed for contact interfaces.

The limit analysis problem is formulated using second order cone programming (SOCP), for which very efficient solution algorithm exists (Andersen et al., 2003), to allow direct modelling of the conic failure surface in sliding for spatial assemblages.

A key aspect of the above mentioned formulation is the capability to take into account non-associative behaviour in sliding according to an iterative procedure that provides accurate and stable solutions with significant CPU savings, if compared to classic approaches, even for problems with a large number of variables. In fact, it was shown that the solution tends to converge towards the corresponding MPEC solution in all the 3D cases examined, but with significantly reduced computational efforts. However, it also means that this formulation might present the same drawback as MPEC of underestimating the actual bearing capacity.

Applications of the developed procedure can be also found in Casapulla et al. (2014) and can be conveniently extended to the identification of failure mechanisms in seismic assessment of rocking masonry structures (Lagomarsino, 2014).

The assumption of infinite compressive strength made in Portioli et al. (2014) follows conventional modelling approaches in rigid block limit analysis of masonry structures, where crushing failure is usually neglected. This assumption is generally suitable to model unreinforced masonry structures with dry-joint or poor quality mortar joints, such as historical buildings, whose behaviour is mainly governed by nil or low tensile strength, sliding and rocking failure.

Nevertheless, crushing could significantly affect the collapse behaviour of masonry structures, as it is the case of strengthened masonry (Orduña and Lourenço, 2003). Indeed, the use of strengthening devices, such as metal ties or composite strips, may produce stress concentration in compression and, as a consequence, crushing failure of masonry panels.

The modelling of crushing failure in rigid block limit analysis has been faced in a number of studies in the literature (Gilbert, 2007).

Among those, it is worth mentioning the three-dimensional formulations proposed by Orduña and Lourenço (2003, 2005a,b). The authors adopted a convex contact model (sometimes also referred to as a 'surface' contact model) and proposed a crushing–hinging criterion corresponding to a quadratic failure condition.

In particular, Orduña and Lourenço (2005b) proposed an alternative solution procedure for the non-associated limit analysis of rigid blocks assemblages, named load-path following solution procedure, which is capable to provide better solutions than minimizing the load factor.

With respect to the focus of the present paper, it is interesting to note that although the existing models provide good agreement between theoretical predictions and experimental outcomes, the use of solution algorithms for the non-linear programs underlying the limit analysis problem might involve long CPU-time as well as numerical stability problems if applied to large scale assemblies.

With that in mind and to take full advantage of the computational efficiency of the SOCP formulation based on the contact point model, in this study the rigid block model developed by Portioli et al. (2014) is extended so as to include an iterative procedure to model the crushing failure as well.

The crushing model takes into account interactions of resultant normal force at interfaces with bending moments and is formulated on the basis of normal forces computed at each contact point. An iterative procedure is developed to linearize the quadratic crushing failure condition.

The linearization of failure conditions – such as the one implemented for crushing – is a well consolidated procedure in the literature (e.g. in Gilbert, 2007).

Nevertheless, it is worth noting that in order to save computational costs in the proposed formulation the iterative solution procedure for crushing runs in combination with the one previously developed for the non-associative behaviour in sliding.

It should also be pointed out that the use of SOCP has already been adopted for the solution of different limit analysis problems, mainly in the field of geotechnical engineering. However, most of these formulations have been developed using associative flow rules and following a continuum based finite element approach (Makrodimopoulos and Martin, 2006, 2007).

Incremental formulations using SOCP for continuum and discrete modelling approaches which take into account non-associative behaviour have also been recently developed (Krabbenhoft et al., 2012). Even so, considering the different field of application, also in these cases crushing is not taken into account.

So the novel contribution of the paper is the computationally efficient solution procedure which has been implemented to take into account both the non-associative friction problem and limited compressive strength using SOCP, with reference to 3D rigid block assemblages interacting through contact interfaces modelled with the point based formulation. Considering that the iterative solution procedures for crushing failure and non-associative sliding run in combination, one of the most important issues of the present study was to investigate the convergence behaviour and accuracy of the proposed coupled procedure.

The paper is organised as follows. The relationships governing the behaviour of the rigid block model and the limit analysis formulation are presented in Sections 2–5. The adopted iterative solution procedure is described in Section 6. In Section 7 a validation study is presented for in-plane and out-of-plane loaded wall panel problems considered in the literature. To assess the accuracy and efficiency of the proposed model when applied to assemblages with a large numbers of blocks and contacts, in Section 8 the formulation is applied to the experimental case study of a full scale masonry pier-spandrel system.

2. The rigid block model

The numerical model is composed of rigid blocks i interacting at contact points k located at the vertexes of the interface j (Fig. 1).

A no-tension frictional behaviour with limited compressive strength is assumed at contact interfaces.

The governing equations are expressed according to Ferris and Tin-Loi (2001), though now extended to three-dimensional block assemblages. The problem is formulated in terms of equilibrium equations, which relate external and internal forces, and of kinematic equations, which ensure compatibility between contact displacement rates and block degrees of freedom.

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