International Journal of Solids and Structures 69–70 (2015) 277–290

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Effective insights into the geometric stability of symmetric skeletal structures under symmetric variations

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ARTICLE INFO

Article history:
Received 16 December 2014
Received in revised form 15 May 2015
Available online 2 June 2015

Keywords:
Mobility
Kinematically indeterminate
Group theory
Initial imperfection
Kinematic constraint
Removal of members

ABSTRACT

Geometric stability is a necessary criterion to guarantee stable equilibrium in engineering structures. However, we generally encounter enormous calculations to examine the geometric stability when we make variations on the geometry or the connectivity of a given kinematically and statically indeterminate structure. This study describes how symmetry is utilized to enhance the mobility and geometric stability analysis of symmetric skeletal structures. Symmetry-extended mobility distinguishes representations of the internal mechanisms and self-stress states from relative mobility based on their inherent symmetries using group-theoretic method. Thus, it acts as an efficient tool to evaluate the order of internal mechanisms that may be indistinguishable by traditional structural approaches. Further, it is used to gain effective insights into the mobility and geometric stability of a symmetric skeletal structure with symmetrically perturbed connectivity or geometry. The first-order changes of symmetry-extended mobility are deduced to describe the changes induced by the variations of nodal coordinates, members, and kinematic constraints, respectively. Examples are given to verify the correctness and effectiveness of the proposed method. We show that the geometry or connectivity of kinematically indeterminate symmetric skeletal structures can be altered while at the same time retaining geometric stability and some or all of the original symmetry. The results have potential application in the design of novel deployable structures.

1. Introduction

Geometric stability is necessary to guarantee stable equilibriums. It is defined as a property of a structure which preserves its geometry under loads and allows the structure to act as a unified system (Macdonald, 2007). Some questions on this topic such as "what conditions are necessary/sufficient for geometric stability?", "What static or kinematic characteristics of a structure will change or remain constant under varied geometries?" attract great attention and interest among researchers. These questions are crucial for many applications in the fields of civil and mechanical engineering, e.g., for developing novel deployable structures or kinematically indeterminate structures.

Exploring answers to the above questions, Maxwell (1864) developed a mobility rule for pin-jointed structures. More recently, Pellegrino and Calladine (1986) classified these structures into four types according to static and kinematic indeterminacy, and proposed a criterion (Calladine and Pellegrino, 1991) for evaluating their geometric stability. Using constraint equations and a statical-kinematic stiffness matrix, Kuznetsov (1991) studied the kinematic mobility and statical possibility of self-stress states, and proposed a criterion for immobility.

Further, most skeletal structures are symmetric (Guest et al., 2010; Wei and Dai, 2010), as they can be transformed into configurations that are physically indistinguishable from the original configuration. Recently, group theory has been utilized as a systematic mathematical tool for studying the stability of symmetric structures (Kaveh and Nikbakht, 2008, 2010; Kettle, 2008; Zingoni, 2009), as well as for designing novel deployable structures based on an existing deployable structure (Sareh and Guest, 2015a,b). These group-theoretic methods not only reduce the computational effort, but also give qualitative benefits and insights (Chen et al., 2014; Zingoni, 2014). Based on the irreducible representations of symmetry groups, Guest and Fowler proposed a symmetry-extended mobility rule for symmetric frameworks (Fowler and Guest, 2000; Guest and Fowler, 2005). Using the symmetry-extended mobility rule, Guest and Fowler (2007)

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http://dx.doi.org/10.1016/j.ijsolstr.2015.05.023
0020-7683/

further identified the symmetries of the internal mechanisms and self-stress states, and thus revealed mobility. Therefore, the geometric stability of some symmetric structures with internal mechanisms can be computed efficiently. The recent examples are illustrated in the cyclically symmetric pin-jointed structures (Chen et al., 2013) and the highly symmetric over-constrained structures (Chen et al., 2012a,b). To provide necessary stability conditions, Connelly et al. (2009) and Chen et al. (2014) used group theory to study the stability of symmetric pin-jointed structures. Zhang et al. (2009) used group theory to investigate the geometric configurations and stability of symmetric tensegrity structures. In addition, group theory can be extended to analyze the mobility and geometric stability of finite mechanisms (Zhao et al., 2009; Ding et al., 2011; Wei et al., 2014; Wei and Dai, 2014) that were explored with screw theory.

As geometric stability has often been evaluated by the positive definiteness of the geometric stiffness matrix, Guest (2006) developed stiffness formulations for prestressed pin-jointed structures. Based on the energy method (Connelly, 1982; Connelly and Whiteley, 1996), Vassart et al. (2000) studied the geometric stability of kinematically and statically indeterminate structures. The reported algorithm is capable of identifying the order of internal mechanisms. Using the principle of potential energy, Kovacs and Tarnai (2009) investigated the equilibrium and geometric stability of bar-and-joint assemblies on the surface of a sphere. Masic et al. (2005) studied the geometric stability of symmetric tensegrity structures with shape constraints. It has been proved that the structural equilibrium is preserved under affine node position transformations. Sultan et al. (2001) formulated the general geometric stability conditions for tensegrity structures. The stability conditions were expressed as a set of nonlinear equations and inequalities on the tendon tensions. Subsequently, Sultan (2013) presented the necessary and sufficient conditions for the exponential stability of prestressable pin-jointed structures, and discussed the advantages of the formulation of the tangent stiffness matrix in analytical manipulations and computations. Meanwhile, some studies have evaluated the geometric stability of a pin-jointed structure by heuristic optimization methods such as genetic algorithms and the ant colony algorithms (El-Lishani et al., 2005; Chen et al., 2012a,b; Koohestani, 2013).

Nevertheless, the above methods usually concern the mobility and geometric stability of a structure with a specific and fixed geometry and connectivity. However, in the preliminary analysis or design process of a structure, the geometry or connectivity might be variable (Zhang et al., 2014). Obviously, repeated calculations for the geometric stability of a structure with variable geometry or connectivity are computationally expensive. Therefore, more efficient numerical methods are required to reduce the relevant computational tasks. Furthermore, it is known that many factors affect the mobility and geometric stability of a structure. Using the singular value decomposition technique, Lu et al. (2007) analyzed the mobility and geometric stability of kinematically indeterminate pin-jointed structures under external loads. They showed that a deployable structure can preserve its geometric stability in certain conditions. Among the components of the stiffness matrices, the main factors affecting the geometric stability of the structure include nodal coordinates, the connectivity patterns of members, and kinematic constraints (Deng and Kwan, 2005; Ohsaki and Zhang, 2006; Chen et al., 2014).

This study explores the impact of symmetric variations on the mobility and geometric stability of symmetric skeletal structures. We proposed a symmetry method that builds on our previous work (Chen et al., 2012a,b; Chen et al., 2014) and the work by Guest and co-workers (Fowler and Guest, 2000; Connelly et al., 2009; Guest et al., 2010). Specifically, we investigate the variations of nodal coordinates, structural members, and kinematic constraints of the structures to provide effective insights into their mobility and geometric stability.

The article is organized as follows. Section 2 introduces the symmetry-extended mobility rule for kinematically indeterminate structures under symmetric variations. Current numerical approaches for evaluating the mobility and geometric stability of a structure are described in Section 2.1. Previous work on the symmetry representations of mechanism modes and self-stress states is presented in Section 2.2. The first-order variations of symmetry-extended mobility for structures with varied connectivity or geometry are derived in Section 2.3. Based on the proposed method, Section 3 presents the impact of the nodal coordinates on the geometric stability of a structure. In the same section, the effect of symmetry migrations is discussed. Sections 4 and 5 demonstrate the impact of the structural members and the impact of the kinematic constraints on the geometric stability of a structure, respectively. Section 6 concludes the paper.

2. Symmetry-extended mobility for structures under symmetric variations

2.1. Mobility of a structure

Maxwell's rule (Maxwell, 1864) is a necessary condition for the mobility of pin-jointed structures by counting structural components. It is valid for kinematically determinate structures; for statically and kinematically indeterminate structures (Pellegrino and Calladine, 1986), Maxwell's rule should be expressed as:

$$m - s = T \cdot j - b - k \tag{1}$$

where T is the magnitude of the rigid-body translation vector, j is the number of all the pin-joints (including boundary nodes), b is the number of members, and k is the number of constraints on the structure (Guest et al., 2010). However, for a free-standing structure (i.e., $k = 0$), k is modified as $k = T + R$ to exclude rigid-body motions, where R is the magnitude of the rigid-body rotation vector (Chen et al., 2014).

In Eq. (1), m is the number of internal mechanism modes, which are the independent vectors in the nullspace of the compatibility matrix $\boldsymbol{J}$, i.e., a solution to the compatibility equation (Pellegrino and Calladine, 1986; Fowler and Guest, 2000):

$$\boldsymbol{Jd} = \mathbf{0} \tag{2}$$

where $\boldsymbol{d}$ is a vector of nodal displacements. Moreover, in Eq. (1), s is the number of self-stress states, which are the independent vectors in the nullspace of the equilibrium matrix $\boldsymbol{H}$, i.e., a solution to the equilibrium equation (Pellegrino and Calladine, 1986):

$$\boldsymbol{Ht} = \mathbf{0} \tag{3}$$

where $\boldsymbol{t}$ is the vector containing the internal forces in the members. Using the virtual work principle, it can be shown that $\boldsymbol{H} = \boldsymbol{J}^{\mathrm{T}}$.

The relative mobility, $m - s$ in Eq. (1), is not sufficient to evaluate the geometric stability of statically and kinematically indeterminate structures. Calladine and Pellegrino (1991) proposed a criterion to identify whether self-stress states can stiffen all the internal mechanism modes. The criterion is equivalent to the positive definiteness of the quadratic form of the geometric stiffness matrix $\boldsymbol{K}_G$ (Guest, 2006; Ohsaki and Zhang, 2006) satisfying

$$\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{K}_G\boldsymbol{M}\boldsymbol{\beta} > 0, \quad \forall \boldsymbol{\beta} \in \Re^m \tag{4}$$

where $\boldsymbol{M}$ is the mechanism mode matrix, and $\boldsymbol{\beta}$ is an arbitrary nonzero vector. Recent work (Deng and Kwan, 2005; Chen et al., 2012a,b; Sultan, 2013) reveals that the criterion provides a necessary condition for the stability of pin-jointed structures. Based on