International Journal of Solids and Structures 69–70 (2015) 305–310

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Stationary dipole at the fracture tip in a poroelastic medium

S.V. Golovin [a,b,*], A.N. Baykin [b]

[a] Lavrentyrev Institute of Hydrodynamics, pr. Lavrentyeva 15, Novosibirsk 630090, Russia
[b] Novosibirsk State University, ul. Pirogova 2, Novosibirsk 630090, Russia

ARTICLE INFO

Article history:
Received 3 November 2014
Received in revised form 16 May 2015
Available online 29 May 2015

Keywords:
Hydraulic fracture
Poroelastic material
Exact solution
Dipole
Filtration

ABSTRACT

This paper presents an analytic self-similar solution describing an equilibrium of a planar semi-infinite hydraulic fracture in a poroelastic medium. The equilibrium of the fracture is sustained by a balance between the pressure of fluid within the fracture and the confining stress due to elasticity of nearby poroelastic material satisfying the Biot model assumptions. Propagation of fluid in the fracture, leakoff through the fracture's walls and filtration in the poroelastic medium towards the fracture's tip form a dipole-type pattern of flow. The solution is used for the verification of a numerical algorithm developed earlier for the pressure transient analysis in water injection wells.

1. Introduction

Hydraulic fractures play an important role in natural processes and engineering practice. Propagation of hydraulic fractures is caused by natural or artificial pumping of large amounts of viscous fluid into the fracture that creates enough pressure on the fracture's walls to withstand and overcome the confining geological stresses. The process of fracture propagation is the interplay of several equally important factors: transport of viscous fluid in a narrow gap of the crack, elastic response of the fracture's walls, filtration of fluid through the walls and interaction with the surrounding pore fluid, breakage of the rock and advance of the fracture's tip. Modelling of the complete picture of fracture dynamics is a challenging problem that is rarely solved in its full statement.

Most often the problem is simplified by taking additional assumptions on the geometry of the fracture, and/or neglecting the pore pressure in the reservoir, and/or prescribing the leakoff of fluid through the walls, etc. Among the most popular, are the 1D models by Perkins, Kern, and Nordgren (PKN) and Khristianovich, Zheltov, Geertsma and de Klerk (KGD), and the generalisation of the latter model to the radial geometry (penny-shaped fractures (Adachi et al., 2007)). The leakoff is often modelled with the use of simplifying hypotheses like the Carter formula (Economides and Nolte, 2000).

Even the simplified models of fracture growth remain nonlinear due to the lubrication approximation of the viscous flow within the crack. This nonlinearity and the singularity of pressure at the fracture's tip cause problems for numerical solution of the models' equations. In this respect it is important to have simple exact solutions that would allow one to check the accuracy and convergence of the numerical algorithm. As for the KGD model, there are a number of analytical and semi-analytical self-similar solutions (Spence and Sharp, 1985; Adachi and Detournay, 2002; Carbonell et al., 1999; Garagash and Detournay, 2005; Garagash, 2006) for different statements of the problem: small or large rock toughness, constant or time-dependent injection rate, viscosity dominated or toughness-dominated regimes. The solutions are used for computation of asymptotics of pressure and fracture opening near the fracture tip, for identification of parameters that distinguish various regimes of fracture propagation, and as benchmarks for numerical simulators (see Mishuris et al. (2012) and Lecampion et al. (2013) and citations therein).

A more reliable mathematical model of a fracture in a poroelastic medium which allows for determination of both pore pressure and elastic rock displacements jointly with the fracture aperture and fracture fluid pressure was developed in Shelukhin et al. (2014). In this model the poroelastic material near the fracture is considered as a homogeneous permeable medium governed by Biot equations (Biot, 1955, 1956). Interaction of the reservoir and fracturing fluid is described naturally within the unified pressure field. The numerical algorithm presented in Shelukhin et al. (2014), based on the finite element method, was numerically tested for convergence, although no comparison with the exact solution was done.

The goal of the present paper is to construct an exact, self-similar solution to the model of a hydraulic fracture in a

* Corresponding author at: Lavrentyrev Institute of Hydrodynamics, pr. Lavrentyeva 15, Novosibirsk 630090, Russia. Fax: +7 (383) 333 1612.
E-mail address: golovin@hydro.nsc.ru (S.V. Golovin).

http://dx.doi.org/10.1016/j.ijsolstr.2015.05.020
0020-7683/

poroelastic medium (Shelukhin et al., 2014) and to demonstrate its applicability for validation of numerical algorithms. The obtained solution describes an equilibrium of a semi-infinite fracture sustained by a dipole-type filtration flow of fracturing and pore fluid in the vicinity of a fracture's tip. The solution is used for verification of the numerical algorithm of paper (Shelukhin et al., 2014) for the pressure transient analysis in water injection wells.

2. Statement of the problem

In this paper we make use of a model proposed in Shelukhin et al. (2014). The mathematical statement of the problem reads as follows. We consider a vertical planar semi-infinite fracture of fixed height $2H$ extending along the positive x-semiaxis with z-axis directed upward along the fracture's tip, see Fig. 1. The fracture is opened in y direction due to the pressure generated by fluid flow inside the fracture. Following Shelukhin et al. (2014) we suppose that fracture's aperture is constant along the vertical coordinate z, so the plain strain approximation is applicable. This implies, that we can limit ourselves to observing only the central cross-section $z = 0$ of the fracture, assuming the 2D model of poroelasticity.

The poroelastic medium is assumed to be isotropic and homogeneous. It is characterised by its porosity ϕ and permeability k, with the solid phase displacement $\mathbf{u}(t, \mathbf{x})$, and the pore pressure $p(t, \mathbf{x})$. Pores are saturated by a single-phase incompressible Newtonian fluid with the effective viscosity η. The linear Darcy law for the fluid velocity $\mathbf{q} = -(k/\eta)\nabla p$ is applicable. We do not distinguish between pore and fracturing fluid, so the viscosity of fluid in the fracture is equal to η. The governing equations of the quasi-static poroelasticity model are the following:

$$\begin{aligned} &\operatorname{div}\tau = 0, \quad \tau = \lambda \operatorname{div}\mathbf{u} I + 2\mu\mathcal{E}(\mathbf{u}) - \alpha p I \\ &S_\varepsilon \frac{\partial p}{\partial t} = \operatorname{div}\left(\frac{k}{\eta}\nabla p - \alpha \frac{\partial \mathbf{u}}{\partial t}\right). \end{aligned} \tag{1}$$

Here $\mathcal{E}(\mathbf{u})$ is the strain tensor $2\mathcal{E}(\mathbf{u})_{ij} = \partial u_i/\partial x_j + \partial u_j/\partial x_i$ $(i, j = 1, 2)$, α is the Biot coefficient, λ and μ are elasticity moduli, and I is the identity tensor. The fluid yielding capacity coefficient S_ε reflects the dependence of the porosity ϕ on $\epsilon = \mathrm{tr}\mathcal{E}$ and p as in Biot (1955):

$$\frac{\partial \phi}{\partial t} = \alpha \frac{\partial \epsilon}{\partial t} + S_\varepsilon \frac{\partial p}{\partial t}.$$

Due to the plane strain approximation, the solid phase displacement vector $\mathbf{u} = (u_1, u_2) = (u, v)$ is two-dimensional, all vector operations are also taken in 2D space of independent variables $(x_1, x_2) = (x, y)$.

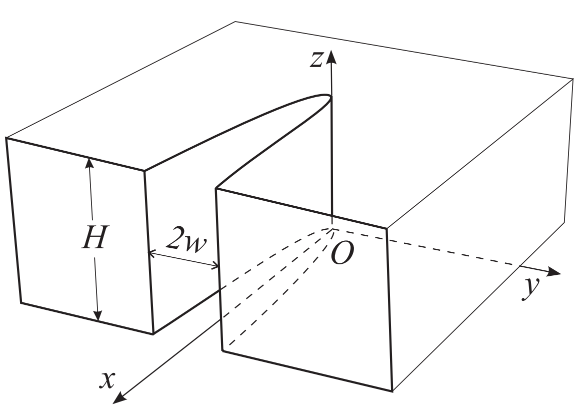

Fig. 1. The geometry of a planar semi-infinite fracture.

Eq. (1) are solved in the domain

$$\Omega = \{(x, y) : |\mathbf{x}| = \sqrt{x^2 + y^2} < R\}.$$

We assume that the fracture is located along x-semiaxis at $y = 0$.

At the outer boundary $\Gamma_R : |\mathbf{x}| = R$ the confining stress σ_∞ is applied and the pore pressure $p = p_\infty$ is prescribed:

$$\Gamma_R : \quad p = p_\infty, \quad \mathbf{n}\cdot\tau\langle\mathbf{n}\rangle = -\sigma_\infty, \quad \mathbf{s}\cdot\tau\langle\mathbf{n}\rangle = 0, \quad (\tau\langle\mathbf{n}\rangle)_i = \tau_{ij}n_j.$$

Henceforth $\mathbf{n}$ is the outer (to the domain Ω) normal and $\mathbf{s}$ is the tangent unit vectors to the boundary; the summation over the repeating index is implied.

The line $y = 0$ is divided into the part $\Gamma_c = \{x \geq 0, y = 0\}$ occupied by the fracture, and the remaining part $\Gamma_s = \{x < 0, y = 0\}$. Outside the fracture on the line $y = 0$ the symmetry conditions (see Shelukhin et al. (2014)) are satisfied:

$$\Gamma_s : \quad \frac{\partial u}{\partial y} = 0, \quad v = 0, \quad \frac{\partial p}{\partial y} = 0. \tag{2}$$

With $S(t, x)$ standing for the fluid pressure inside the fracture, the force balance over the fracture's wall yields

$$\Gamma_c : \quad p = S, \quad \mathbf{n}\cdot\tau\langle\mathbf{n}\rangle = -S, \quad \mathbf{s}\cdot\tau\langle\mathbf{n}\rangle = 0. \tag{3}$$

Here we neglect the tangential stress due to the fluid friction on the fracture's walls in comparison with the normal stress.

The fluid flow in the fracture is governed by the mass conservation law complemented with the Poiseuille formula:

$$\frac{\partial w}{\partial t} + \frac{\partial (wq)}{\partial x} = -q_l, \quad w \equiv v|_{y=0}, \quad q = -\frac{(2w)^2}{12\eta}\frac{\partial S}{\partial x}. \tag{4}$$

Here $2w$ is the fracture's aperture, q is the fluid velocity in the x-direction. No fluid lag is assumed at the fracture tip.

The leakoff velocity q_l is given by the Darcy law as

$$q_l = -\frac{k}{\eta}\frac{\partial p}{\partial y}\bigg|_{y=0}. \tag{5}$$

The resulting equation governing the flow inside the fracture reads

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial x}\left(\frac{w^3}{3\eta}\frac{\partial S}{\partial x}\right) + \frac{k}{\eta}\frac{\partial p}{\partial y}\bigg|_{y=0}. \tag{6}$$

The fluid flow rate (per unit height) along the fracture at a fixed section $x = x_0$ is computed as

$$Q = -\frac{w^3}{3\eta}\frac{\partial S}{\partial x}\bigg|_{x=x_0, y=0}. \tag{7}$$

The value of Q is usually prescribed and serves as one of the governing parameters of the entire process. Note that formula (7) as well as the Poiseuille formula (4) are valid only for the Newtonian incompressible fluid.

Eq. (6) is often referred to as the lubrication theory equation (Adachi et al., 2007). Note that, due to the right-hand side (5), Eq. (6) represents a boundary condition for equations of the main model (1). The leakoff velocity q_l is obtained here naturally in the course of the problem's solution, which differentiates the model favourably from the usual artificial approximations like Carter's formula or other similar expressions (Economides and Nolte, 2000). However, the nonlinearity of the lubrication Eq. (6), makes the whole model nonlinear.

For computational reasons it is convenient to homogenise the conditions on the outer boundary Γ_R. It can be done by substraction of the homogeneous solution corresponding to the compression of the layer without a fracture by the confining stress and the pore pressure at infinity:

$$\tilde{\mathbf{u}} = \mathbf{u} - \varkappa\mathbf{x}, \quad \tilde{p} = p - p_\infty, \quad \varkappa \equiv \frac{\alpha p_\infty - \sigma_\infty}{2(\lambda + \mu)},$$