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Axisymmetric problems of a penny-shaped crack at the interface of a bi-material under shear and compression

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ABSTRACT

This paper studies an axisymmetric problem of a penny-shaped interface crack of a bi-material subjected to compression and radial shear at the crack surfaces in cylindrical coordinate system. The Hankel transform technique is applied to convert the problem to dual integral equations. Closed-form solutions for mode-I and II stress intensity factors are obtained for arbitrary radial shear loading at the crack surfaces. Obtained results indicate that no oscillation singularity occurs. Explicit expressions for the stresses and displacements in the whole bi-material elastic space are derived for constant and linear radial shear stress acting on the crack surfaces, respectively. Under uniform radial shear stress, normal and shear stresses have a usual square-root singularity behind and ahead of the crack front, respectively, and the induced normal stress exhibits a logarithmic singularity at the crack center. Under linearly-distributed radial shear stress, the logarithmic singularity at the crack center disappears, and the square-root singularity is still present for the normal and shear stresses near the crack front. The induced mode-I stress intensity factors depend on Dundurs' parameter and on applied shear loading. Numerical results for a penny-shaped interface crack in Al_2O_3/PMMA bi-material are presented to show the influence of the bi-material properties on the stress distribution.

1. Introduction

When two dissimilar materials are bonded together, the properties of interface play a crucial role in assessing the structural safety and integrity. Due to deformation mismatch of dissimilar materials with different material properties, stress concentration at the interface occurs and the interface is one of potential crack initiation sites. Therefore, interface crack problems are an important subject of researches for better understanding mechanical behaviors of two bonded dissimilar materials. Up to date, there are still inherent challenges in characterizing the stress singularity of the crack tip or front.

In theory, the interface crack problems have received much attention of researchers from the study of Williams (1959), who firstly discovered the oscillatory singularity near an interface crack tip. Later, a large amount of research work has been reported and various methods have been formulated to treat the elastic problems of interface cracks. Owing to lack of physical meaning of oscillatory singularity near an interface crack tip, the contact model of crack surfaces near the tip has been formulated to eliminate the oscillatory singularity (Comninou, 1977, 1978; Gautesen and Dundurs, 1987, 1988).

Most researches focus on a tensile crack, which is one of the most-frequently studied situations. As well known, closed crack growth under compressive loading is also often encountered in nature. For example, some phenomena on failure of crystalline solids such as ice, rock, silicate minerals, etc. due to compressive loading are usually observed. In particular, catastrophic earthquake may be created by propagation of strike–slip and thrust faults due to confining pressure of the ambient structure arising from change in temperature and it may be understood as a consequence of crack growth under shear and compression. Research work on closed crack advance in a homogenous isotropic medium under compressive loading has been conducted (Ashby, 1986). For a closed crack in compression, a theoretical negative mode-I stress intensity factor has no sense from a physical point of view and cannot be used to predict crack growth (Li et al., 2012). In this case, two elastic T-stresses are present at a crack tip and they in connection with mode-II stress intensity factor commonly affect crack kinking, apparent fracture toughness and small-scale yielding zone (Li and Xu, 2007; Li et al., 2009). Makaryan et al. (2011) also

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analyzed a cracked elastic layer under compressive loads and found that the crack-tip field is only dominated by mode-II singularity and mode-I stress intensity factor vanishes. For an interface crack under compressive forces, Qian and Sun (1998) employed the contact zone model to study a frictional interface crack under shear and compression, in which the oscillatory singularity at the crack tip may be avoided. Dorogoy and Banks-Sills (2004) utilized a finite difference method to cope with shear loaded interface crack with influence of friction. Correa et al. (2008) gave a numerical analysis of crack growth at the fiber–matrix interface under compression. Fracture characterization of an interface crack with frictional contact of the crack surfaces has also been dealt with in Kim and Lee (2009).

The above-mentioned studies focus on a two-dimensional elastic analysis relative to interface cracks under shear and compression. For a three-dimensional bi-material elastic space with a two-dimensional penny-shaped crack, an oscillatory singularity near the crack front has been shown for elastic stress fields (Willis, 1972; Kassir and Bregman, 1972; Lowengrub and Sneddon, 1974). Some numerical approaches for determining stress intensity factors of a three-dimensional cracked solid are based on the oscillatory singularity (Nagai et al., 2007; Graciani et al., 2009). For a penny-shaped inclusion at the interface of two bonded dissimilar materials, an oscillatory singular behavior can also be found (Selvadurai, 2000; Li and Fan, 2001). As we know, the oscillatory singularity for an interface crack is undesirable, and many models have been proposed to eliminate it. Similar to an analysis of a cracked elastic plane (Comninou, 1977), Keer et al. (1978) extended the contact zone model to the three-dimensional elasticity problem for an interface penny-shaped crack and assumed an annular frictionless contact zone at the crack circumference, which eliminates the unmeaning oscillatory behavior of stress near the crack front. Relevant work on a penny-shaped crack at the interface of a bi-material under compressive force and shear is very limited. Nevertheless, an elastic analysis of a homogeneous material with a penny-shaped crack under radial shear can be found in Kassir and Sih (1975) and Lee (2013). Also, Nazarenko et al. (2000) analyzed the influence of initial stress on fracture of a half-space containing a penny-shaped crack under shear. Recently, Li et al. dealt with an interface crack of two bonded dissimilar elastic half-planes (Li et al., 2015a) and half-spaces (Li et al., 2015b) under shear and compression and found no oscillatory singularity near the crack tips or front.

The purpose of this paper is to study an interface penny-shaped crack of a bi-material subjected to shear and compression. Two cases are dealt with. One is a uniform radial shear at the crack surfaces and the other is a linearly-distributed radial shear stress at the crack surfaces. For these two typical cases, the problem is solved by use of the Hankel transform technique. Full elastic fields at any position of a bi-material are obtained in explicit form. Obtained results indicate that the crack front is dominated by the usual square-root singularity, rather than the oscillatory singularity. Applied shear stress can induce singular normal stress at the crack surface behind the crack front and singular shear stress ahead of the crack front. The normal stress may exhibit a logarithmic singularity, depending on applied shear.

2. Statement of the problem

Consider a penny-shaped crack lying at the interface of two bonded dissimilar materials under radial shear stress and normal stress at the crack surface. These stresses may be originated by applied loading such as a compressive force acting at the central axis, as shown in Fig. 1. The crack is located at the circular region in the *xoy*-plane, i.e. $r \leqslant a, z = 0$, where (r, φ, z) is a system of cylindrical polar coordinates with the origin is at the crack center. Here two bonded dissimilar elastic solids are assumed to occupy in the upper and lower half-spaces, respectively. From the viewpoint of fracture mechanics, of much interest is the singular elastic behavior induced by a crack. In the following, an axisymmetric problem is considered for simplicity in the present study and associated boundary conditions are stated as follows

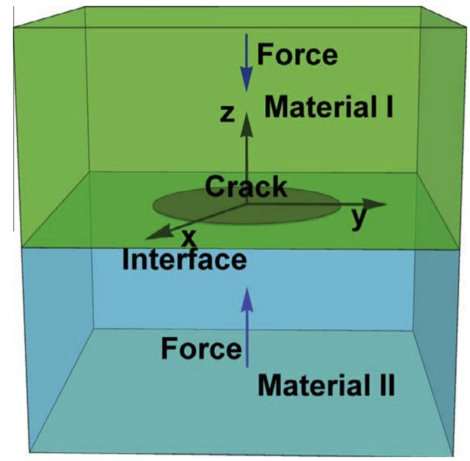

Fig. 1. Schematic of a penny-shaped crack at the interface of a bi-material elastic space.

$$\sigma_{rz}^{\mathrm{I}}(r,0^+) = \sigma_{rz}^{\mathrm{II}}(r,0^-) = -\tau_0(r), \quad r < a, \tag{1}$$

$$\sigma_{rz}^{\mathrm{I}}(r,0^+) = \sigma_{rz}^{\mathrm{II}}(r,0^-), \quad u^{\mathrm{I}}(r,0^+) = u^{\mathrm{II}}(r,0^-), \quad r > a, \tag{2}$$

$$\sigma_{zz}^{\mathrm{I}}(r,0^+) = \sigma_{zz}^{\mathrm{II}}(r,0^-), \quad w^{\mathrm{I}}(r,0^+) = w^{\mathrm{II}}(r,0^-), \quad r \geqslant 0, \tag{3}$$

$$\sigma_{rz}(r,z) \to 0, \quad \sigma_{zz}(r,z) \to 0, \quad r^2 + z^2 \to \infty, \tag{4}$$

where $\tau_0(r)$ is a prescribed radial shear stress at the crack surface with reference to cylindrical coordinate system, which can be determined by solving an elastic stress field without crack, u and w are the displacement components along the radial and axial direction, respectively, σ_{rz}, σ_{zz} are radial and axial stress components, and a quality with superscripts I and II specifies the one in materials I (upper) and II (lower), respectively.

The crack surfaces are assumed to be smooth and in contact each other under compressive loading. This directly leads to that at the crack surfaces, the normal stress and axial displacement must be continuous. Furthermore, at the bonding part of the interface, the normal stress and axial displacement are obviously continuous. Thus we have the conditions in (3) for the entire interface $z = 0$. In addition, due to free shear stress at the crack surfaces, when superposing a nonsingular elastic field, the conditions in (1) at the crack surfaces can be given. The conditions in (2) describe the continuity of shear stress and radial displacement for the crack-free part at $z = 0$.

It is particularly pointed out that the boundary conditions for a penny-shaped interface crack studied in this paper are different from those for the classical shear interface crack. For the latter case, the normal stress at the surfaces of an interface crack is prescribed, i.e. $\sigma_{zz}^{\mathrm{I}}(r,0^+) = \sigma_{zz}^{\mathrm{II}}(r,0^-) = -\sigma_0(r)$ for $r < a$ is required. Nonetheless, due to the sliding contact of the crack surfaces for a bi-material in compression, we replace the above assumption with $\sigma_{zz}^{\mathrm{I}}(r,0^+) = \sigma_{zz}^{\mathrm{II}}(r,0^-)$ and $w^{\mathrm{I}}(r,0^+) = w^{\mathrm{II}}(r,0^-)$. This also connotes that owing to contact of two crack surfaces, the normal stress is no longer stress-free when solving the corresponding crack problem. It is this essential difference that gives rise to a usual square-root singularity, rather than an oscillatory singularity, for shear stress near the crack front, as will be seen in the following analysis.