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Analytical investigation of structurally stable configurations in shape memory alloy-actuated plates

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A B S T R A C T

Strains produced by active materials embedded in plates have been extensively used to manipulate the shape of surface-like engineering structures. Shape memory alloys (SMAs) are active materials that provide a significant amount of strain under large stresses, a characteristic of great utility in such morphing structures. In this work, an analytical approach to approximate the deformation of plates with SMA constituents is developed via the Rayleigh–Ritz method. An additive set of kinematically admissible displacement fields with unknown coefficients is used to describe the plate displacement field. The total potential energy is then calculated using the displacement field, loading conditions, and constitutive relations for the plate layer(s) composed of SMA wire meshes, dense SMA films, and/or elastic material. The unknown coefficients are then found via minimization of the total potential energy. This approach provides closed-form expressions for the approximate deformation of the plates including multistable configurations. The response of circular SMA-based plates is studied herein. The results show that temperature fields with a linear variation in the radial direction induce multistable configurations in which the plate Gaussian curvature is determined by the direction of the temperature gradient. An alternative inversion of the proposed approach is used to directly compute the temperature field required to morph a plate towards a prescribed goal shape. The obtained closed-form expressions show good agreement with detailed non-linear finite element analysis simulations. The proposed approach provides analysts with a low computational cost and relatively simple implementation to assess the potentially stable configurations of SMA-based plates under given loading conditions. Knowledge of such stable configurations is very valuable in the design of SMA-based morphing structures.

1. Introduction

Manipulating the shape of surface-like structures (e.g., plates) and conceiving morphing structures is an active research subject in engineering (Seffen and Maurini, 2013; Fernandes et al., 2010). *Active materials*, those that convert various forms of energy into mechanical work (Srinivasan and McFarland, 2001; Leo, 2008), provide the means to induce controllable inelastic (usually recoverable) strains that can exert such manipulations. The design of morphing surface-like structures with intrinsic material actuation is a challenging problem. It entails two main difficulties: (i) taking into account non-linear material and geometrical effects when designing the structure; and (ii) conceiving appropriate and especially efficient actuation techniques to morph the structure among different goal shapes.

For example, the art and theory of origami have provided numerous solutions to the aforementioned challenges during the past decades (Lang, 2007). Origami provides novel methods to assemble and morph structures (Bassik et al., 2009; Fernandes and Gracias, 2012; Tolley et al., 2014; Kuribayashi et al., 2006; Pandey et al., 2013; Cheng et al., 2013; Peraza-Hernandez et al., 2013b,f, etc.). Furthermore, *self-folding structures*, those that have the capability of folding and/or unfolding among different shapes without requiring externally applied mechanical loads, have been developed by leveraging the use of active materials as agents of fold generation (Peraza-Hernandez et al., 2014c).

The mechanism for morphing these origami-based structures has been primarily *folding* (i.e., *bending*). Bending deformation is isometric and conformal with respect to the neutral surface of a plate structure (i.e., there are no changes in lengths and angles along such a surface resulting from this deformation mode). Deformation via bending provides for large global deflections and rotations; however, such a deformation mode is based primarily

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on disallowing changes in the *Gaussian curvature* of the structure. Gaussian curvature is the product of the two principal curvatures at any region of a surface ($g = \kappa_I \kappa_{II}$) (Calladine, 1989) where the *principal curvatures* κ_I and κ_{II} correspond to the maximum and minimum values of the local curvatures (Pressley, 2010). Such principal curvatures are oriented orthogonally to each other and aligned in the principal directions $\boldsymbol{v}_{\kappa_I}$ and $\boldsymbol{v}_{\kappa_{II}}$, respectively. Considering a surface subdomain sufficiently small so as to be effectively approximated by a surface patch of uniform Gaussian curvature, four cases can be distinguished: (i) if both κ_I and κ_{II} are zero, the region is *planar* and has zero Gaussian curvature (Fig. 1(a)); (ii) if either κ_I or κ_{II} is zero and the other is non-zero, the region is *parabolic* and has zero Gaussian curvature (as in a bend; Fig. 1(b)); (iii) if κ_I and κ_{II} are both < 0 or both > 0, the region is *elliptic* and has positive Gaussian curvature (Fig. 1(c)); and (iv) If κ_I and κ_{II} are both non-zero and of opposite sign, the region is *hyperbolic* and has negative Gaussian curvature (Fig. 1(d)), Pressley, 2010. Deformation via bending (not allowing changes in Gaussian curvature) largely limits the spectrum of shapes that can be obtained by a structure. Furthermore, regions of non-zero Gaussian curvature from initially planar structures are often needed for aerodynamic, aesthetic, or load bearing reasons.

One approach for the achievement of changes in Gaussian curvature in plate structures is through the generation of inhomogeneous *in-plane extensional/shear strain* fields throughout their middle surface (Seffen and Maurini, 2013; Fernandes et al., 2010; Yu et al., 2000; Liu et al., 2004; Ryu and Shin, 2006). Relatively moderate to large rotations in plate structures entail a coupling between bending and in-plane extension/shear. For thin plate structures, there is potentially a choice for engineers in the *mode of actuation*, the so-called "actuation paradigm" mentioned by Modes et al., 2011: whether in-plane extensional/shear actuation can provide a feasible alternative to bending actuation. The usage of in-plane extensional/shear strains and bending strains (i.e., those varying linearly through the plate thickness), either separately or in combination, can provide for an extensive set of possible shapes that can be attained by a plate structure (Seffen and Maurini, 2013).

Shape memory alloy (SMA)-based plate structures are considered herein. Shape memory alloys are active materials that undergo solid-to-solid phase transformations induced by appropriate temperature and/or stress changes during which they can generate or recover seemingly permanent strains (Lagoudas, 2008). These characteristics allow them to have several existing and potential applications in diverse fields such as aerospace (Hartl and Lagoudas, 2007), biomedical (El Feninat et al., 2002), and others (Mohd Jani et al., 2014). Shape memory alloys exhibit the highest actuation energy density of all active materials. This indicates that SMAs can provide a significant amount of strain under large stresses, a characteristic of great utility in morphing structures for use in realistic conditions.

A schematic of the stress–temperature phase diagram for SMAs is shown in Fig. 2. It can be described by the transformation temperatures at zero stress: martensite start M_s, martensite finish M_f, austenite start A_s, and austenite finish A_f. The slopes of the transformation boundaries in the stress–temperature hyperspace are called the stress influence coefficients C^M and C^A (Lagoudas, 2008)[1]. For the present constitutive model described in detail in Section 2.3, the evolution of the martensite volume fraction is determined via a transformation function (analogous to a yield function in classical plasticity) that depends on stress and temperature (see Eq. (15)). Such a transformation function captures the essential features of the phase diagram shown in Fig. 2. The form of the transformation function is selected depending on whether the SMA is transforming from austenite to martensite (forward transformation) or from martensite to austenite (reverse transformation). *Transformation strains* associated with the evolution of the martensite volume fraction may be generated during forward transformation and are recovered during reverse transformation.

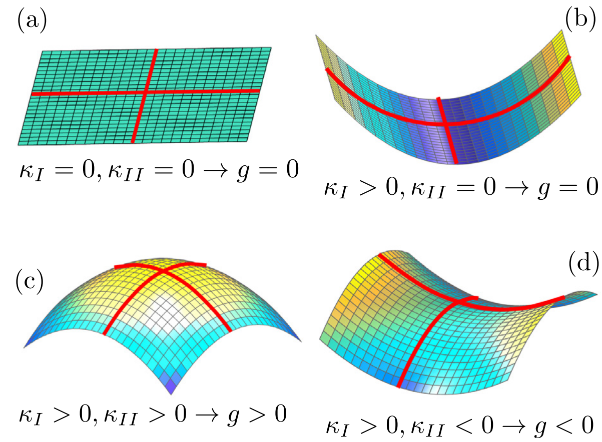

Fig. 1. Principal curvatures and Gaussian curvature of different surfaces.

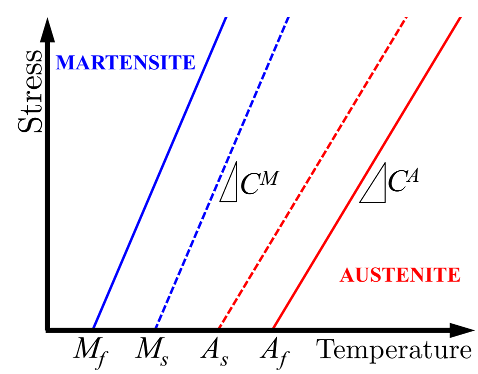

Fig. 2. Schematic of an SMA stress–temperature phase diagram.

For the present application, SMA domains initially in stress-free oriented martensite states are considered (i.e., having pre-generated transformation strains, denoted as pre-strains in this work). During reverse transformation, which is the case considered herein, the direction and magnitude of the transformation strain evolution are defined such that all transformation strains existing at the cessation of forward transformation (the pre-strain in the cases considered here) are fully recovered when the SMA transforms fully back into austenite (see Eq. (13)). Thus, by heating the SMA and moving from the austenite start boundary towards the austenite finish boundary in the stress–temperature space (Fig. 2), the pre-strains in the SMA are recovered and actuation is produced.

The SMA-based plate layups considered in this work are shown in Fig. 3. The plates may be comprised of a single SMA film that is pre-strained homogeneously in the in-plane directions (Fig. 3(a)). The film can provide half of the maximum recoverable uniaxial strain by the SMA (denoted H) in any two orthogonal directions in the plane co-planar to the plate (as shown in Fig. 3(a)). The plates may also be comprised of a three-layer laminate with two outer layers of SMA wire meshes separated by a middle compliant

[1] Generally, the slopes of the transformation boundaries in the stress–temperature phase diagram are not constant and thus measured at a reference stress level (Lagoudas et al., 2012).

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