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Application of the Virtual Fields Method to the uniaxial behavior of rubbers at medium strain rates

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ABSTRACT

The Virtual Fields Method (VFM) is used to explore strain-rate dependent mechanical properties of a hyperelastic material. In this method, the principle of virtual work is constructed to inversely obtain the Young's modulus and Poisson's ratio of a given material from optical measurements of displacement obtained during a dynamic loading event. The virtual displacement field is designed so that acceleration fields, and thereby inertial forces, are used to calculate the material properties, and the traction force term in the principle of virtual work can be eliminated. Experimentally, this means that no force measurements are required during dynamic loading. Prior to the experimental investigations, a simple analytical calculation and finite element model were used in order to simulate the method; the output from the VFM showed good agreement with the given material coefficients. For the experimental work, pure silicone rubber was chosen as a model material. This rubber was tested in tension using a drop-weight apparatus at a medium strain rate (c.a. 160 s^{-1}), using high speed photography and Digital Image Correlation to provide strain and acceleration data which were subsequently analyzed by use of the VFM. By using static pre-stretching prior to the dynamic load, the hyperelastic behavior can be investigated up to large strains, even though the dynamic loading itself only has a small strain amplitude. By optimizing the differential one-term Ogden model to modulus estimations at each of the pre-stretching locations, the nonlinear stress–strain curves were reconstructed. The initial modulus change between these dynamic experiments and quasi-static tests was compared to the storage modulus increment obtained from DMA tests on the same material.

1. Introduction

The inverse method is an approach by which physical parameters of materials are inversely characterized by observing their physical behavior (Avril et al., 2008). In mechanical characterization, the inverse method is used to seek unknown material coefficients, e.g. stiffness, using observed experimental data, e.g. force and strain, through a given constitutive relation. One of the popular inverse methods is finite element model updating where material parameters are optimized by minimizing the difference between numerical and experimental data. This minimization usually requires the use of an iterative method. However, another inverse method, the Virtual Fields Method, VFM, (Pierron and Grédiac, 2012) is a non-iterative approach able to directly obtain material coefficients when an unknown parameter is linearly dependent on an experimental observation. This non iterative method brings two advantages: faster computational time and no influence from the initial parameter estimates. In this research, the VFM is applied to characterize the dynamic behavior of silicon rubber under medium strain rates using a simple drop-weight apparatus and high speed imaging.

Elastomers are used in many applications due to their good damping properties and mechanical softness. The large energy dissipation capability is widely utilized in high strain-rate conditions, for example impact resistance (Davidson et al., 2004), in which the strain rate can be higher than 100 s^{-1}. When deformed at such high rates of strain, elastomers show a significant change in mechanical properties, compared to those under quasi-static loading, due to their viscoelastic behavior (Sarva et al., 2007). Accurate characterization of these mechanical properties is essential for the reliable and effective use of elastomers over a wide range of strain rates.

The mechanical characterization of elastomers at low strain rates (e.g. 10^{-3}–10 s^{-1}) can be conducted by various testing methods, e.g. uniaxial and biaxial stretching (Ogden, 1972). Dynamic tests for elastomers are not straightforward, owing to several difficulties caused by their very low stiffness. Firstly, the stress wave speed is low; hence if a rapid deformation is introduced into one

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end of the specimen, the time taken to reach static stress equilibrium is significant compared to the duration of the experiment (Song and Chen, 2005). The long duration of the non-equilibrium state produces difficulties in measuring forces in the specimen in a high rate experiment, and also in interpreting experimental data in order to extract material properties. In compression, this problem could be avoided by using a specimen which has a large radius to thickness ratio (Shergold et al., 2006; Shim and Mohr, 2009). However, this large ratio can cause problems such as interface friction, lateral inertia and poor accuracy in the strain measurement (Chen and Song, 2011). One of the standard techniques for high rate compression testing is the split Hopkinson bar (SHPB). Chen et al. (2002) showed that the use of a thin specimen cannot solely achieve a good stress equilibrium state in an SHPB experiment; but pulse-shaping techniques should be simultaneously used so that the rise time of the incident pulse is longer and smoother. Considering the deformation of the specimen, pulse shaping gives two advantages. Firstly, the initial strain rate is reduced, meaning that less strain is induced in the specimen before mechanical equilibrium is achieved; secondly, the less impulsive loading does not induce structural vibrations in the specimen.

Dynamic tensile tests on elastomers are even more challenging than compression: the achievement of stress equilibrium is more difficult due to the specimen design which inevitably requires a large length to thickness ratio. The specimen should have at least a certain minimum length in order to experience uniaxial stress and avoid any other stress states, not only triaxial but also planar or biaxial, in which elastomers exhibit a different material behavior. The achievement of static stress equilibrium is especially difficult during the initial stages of loading; this is compounded by the fact that inaccurate force measurements due to small force signals during the initial loading can make confirmation of equilibrium difficult. These problems become more severe as the strain rate increases. A number of authors have conducted dynamic tensile tests on elastomers with some modifications to the traditional split Hopkinson tension bar (SHTB). Cheng and Chen (2003) used a short specimen and pulse shaping technique for an SHTB test on EPDM rubber at a strain rate of about 3000 s^{-1}. Comparison of the stress–strain curves of the dynamic and quasi-static tests showed significant rate dependency. A similar experiment was conducted on polyurethane using a traditional SHTB system in combination with a pendulum striker (Kanyanta and Ivankovic, 2010). Nie et al. (2008) developed a special clamping system in an SHTB in order to use a thin tubular shape specimen with a very short gauge length of 1 mm. The authors found that this specimen design enabled to reduce lateral and longitudinal inertia effects. Apart from the conventional Hopkinson bar technique, Roland et al. (2007) developed a special drop-weight test apparatus in which a tensile load was applied at both ends of a polyurea specimen. In this study, stress equilibrium was confirmed by the similarity of the forces measured at each end of the specimen.

In contrast to the conventional high strain-rate tests described above, the use of the Virtual Fields Method does not require static stress equilibrium during the dynamic experiment; instead, the non-equilibrium state is used as a measurement. In particular, by measuring specimen displacement fields as the wave propagates through a specimen, acceleration field data can be obtained which replaces the need for force measurements. Recently, the VFM was applied to a high strain-rate test on glass (Moulart et al., 2011), carbon fibre reinforced epoxy composites (Pierron et al., 2014) and aluminium (Pierron et al., 2010) using a conventional Hopkinson bar system. Instead of using force signals from the bar, full-field kinematic data (strain and acceleration fields) measured by high-speed imaging were implemented for material parameter characterization with a mathematical manipulation of the principle of virtual work. A similar mathematical procedure is described in the analytical section of this paper. The use of acceleration fields is particularly advantageous for dynamic tests on elastomers because high-speed full-field measurement is easier to implement: owing to the low speed of stress wave propagation a moderate imaging speed can be chosen so that good quality images are obtained (Pierron et al., 2011). Also, the measurement of non-equilibrium state can significantly simplify the experimental complications discussed above, in particular those required to ensure stress equilibrium in these soft materials under dynamic loading.

An experimental challenge in the use of the VFM on hyperelastic materials lies in the introduction of large amplitude stress waves: large deformation speeds are required and the non-linearity in the material behavior can lead to the formation of a shock wave, which prevents measurement of the acceleration field. Another approach to considering non-linear behavior over a large strain range is therefore explored in this paper. Here, a pre-stretching method is developed, in which a specimen is statically preloaded to a range of fixed strains and then dynamically loaded with small amplitude stress waves; dynamic test data at different strain locations are simultaneously used to reconstruct a stress–strain curve over a large range of strain. A similar approach has previously been employed in the literature, in which a pre-stretch was applied up to the strain hardening region in which a shock wave is propagated in a rubber specimen (Niemczura and Ravi-Chandar, 2011). In this paper, the analysis data, strain, were collected from a central line along the specimen and in the loading direction so that one-dimensional jump conditions could be used to obtain the stress in the shock wave region. In the present work, the application of a material constitutive relation in the principle of virtual work equation enables the use of full-field data so that any experimental or analytical limitations raised by the assumption of one-dimensional shock wave propagation can be overcome.

An alternative characterization method for elastomers is dynamic mechanical analysis (DMA) (Mott et al., 2011), which may be used in conjunction with time temperature superposition (Williams et al., 1955); however its analysis is limited to a small strain range, e.g. 0.1%. In spite of this limitation, DMA tests on elastomers provide useful information which can help validate the initial material behavior at a similar strain rate obtained from a large-scale dynamic test. In this study, a DMA test on silicone rubber in a tensile configuration was used for comparison with the VFM analysis.

This paper explores the application of the VFM to experiments on silicone rubber at medium strain rates in tension. Firstly, a simple theoretical calculation is presented explaining how to implement the VFM for a non-equilibrium state in a uniaxial loading configuration. Then, non-equilibrium states in linear-elastic, hyperelastic and visco-hyperelastic materials were produced using finite element simulation (ABAQUS/explicit) in order to test the capability for parameter estimation of the VFM and pre-stretching method. Identical analysis procedures were then applied to experimental data obtained from silicone rubbers dynamically loaded in uniaxial tension by a simple drop-weight test apparatus. This experimental procedure produced a number of modulus estimations, one at each pre-stretching location. The material parameters, μ and α, of the one-term Ogden model (Ogden, 1972) were obtained by means of optimizing the differential form of the model with respect to the modulus estimations from the VFM and the static stress–strain data. This model was chosen based on its ability to describe quasi-static data obtained on the silicone elastomer, but other constitutive models could be used. Finally, the ratio of the initial modulus between the dynamic (the VFM and drop-weight test) and static tests was compared to the storage modulus ratio obtained from a master curve produced by dynamic mechanical analysis on the same material.

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