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Non-linear elastic micro-dilatation theory: Matrix exponential function paradigm

Hamidréza Ramézani [a,*], Jena Jeong [b,1]

[a] École Polytechnique de l'Université d'Orléans, Université d'Orléans, ICMN, UMR CNRS 7374, Interfaces, Confinement, Matériaux et Nanostructures, 8 rue Léonrad de Vinci, 45072 Orléans, France

[b] Université Paris-Est, Institut de Recherche en Constructibilité, ESTP, 28 Avenue Président Wilson, 94234 Cachan Cedex, France

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ABSTRACT

In the present paper, the micro-dilatation theory or void elasticity is extended to both large displacement and large dilatation. Firstly, the deformation gradient tensor has been freshly defined by means of the matrix exponential function. The newly defined relation for the deformation gradient has painstakingly investigated for the uniqueness, decomposition issues as well as objectivity and isotropy considerations. The relation of the displacement gradient and deformation gradient tensor is brought via the matrix logarithm function. The micro-dilatation theory constitutive laws are derived using the thermodynamic principles under the zero-centrosymmetric, weakly-centrosymmetric and fully-centrosymmetric cases. These cases have been derived and scrutinized by the numerical experiments. To achieve this assignment, the basic loadings are taken into account, e.g. the hydrostatic loading, simple traction and shear. Some conclusions and outlook pertaining to the above-mentioned cases and variable bulk density have thereafter discussed.

1. Introduction

1.1. Historical overview on the micro-dilatation theory

The micro-dilatation theory belongs to the generalized continuum family. The generalized continuum mechanics takes advantage of the additional state field variables besides the displacement vector, $u \in \mathbb{R}^3$. The choice of the state field variables sustains various theories, e.g. micro-morphic theory (Eringen, 2001), micro-strain theory (Forest and Sievert, 2006), micro-stretch theory (Eringen, 2001), micro-polar or so-called Cosserat theory (Toupin, 1962; Toupin, 1964; Eringen and Suhubi, 1964; Mindlin, 1964; Eringen, 2001), micro-dilatation theory (Nunziato and Cowin, 1979; Cowin and Nunziato, 1983) and couple stress theory (Mindlin and Tiersten, 1962; Mindlin, 1963; Toupin, 1962) (see Appendix A). As pointed out earlier, the micro-dilatation theory was initially proposed in the early eighties by Nunziato–Cowin's paper (Nunziato and Cowin, 1979) and it was followed in Cowin and Nunziato (1983, 1984b,a); Passman (1984) and Puri and Cowin (1985), Cowin (1985) for the plane waves and visco-elastic behavior. The micro-dilatation was also studied in the literature at the late 1980's, e.g. Chandrasekharaiah (1987). There is a time gap for the micro-dilatation theory or so-called void elasticity between 1990 and 2000. The most outstanding works in revival of the micro-dilatation can be addressed in Markov (1995), Inan and Markov (1995), Scarpetta (2002), Ciarletta et al. (2003), Dey et al. (August 2004).

Iovane and Sumbatyan utilized the micro-dilatation for the dynamic problem of the concentration of stresses near the edges of a crack located in a porous elastic space (Iovane and Sumbatyan, 2005). Iovane and Nasedkin performed the 2D-FEM solutions for the elastic-porous bodies in Iovane and Nasedkin (2005). Some relevant studies pertaining to the application of micro-dilatation theory to the wave propagation and the numerical implementations can be also addressed in Iovane and Nasedkin (2009, 2010a,b). The other relevant works in conjunction with the micro-dilatation are also available in Birsan (2003), Birsan (2006) Chirita et al. (March 2006), Chirita and Ghiba (2010a,b), Singh (2011) and lately in Ramézani et al. (2012b); Thurieau et al. (2013), Jeong et al. (2013b), Thurieau et al. (2014).

* Corresponding author. Tel.: +33 2 38 257879; fax: +33 2 38 255376.
E-mail addresses: hamidreza.ramezani@univ-orleans.fr, hamidreza.ramezzani@gmail.com (H. Ramézani), jjeong@adm.estp.fr, jena.jeong@gmail.com (J. Jeong).
[1] Tel.: +33 1 49 082303; fax: +33 1 45 476039.

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Nomenclature

Constants

α void diffusion coefficient in [N]

β void coupling modulus in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

λ first Lamé's coefficient in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

μ second Lamé's coefficient in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

μ_c Cosserat coupling modulus in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

ν Poisson's ratio in [–]

ω micro-dilatation visco-elasticity modulus in [Pa.s]

ρ_{R} bulk density at reference configuration in $\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$

$\mathbf{G}$ shear modulus in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

ξ void stiffness modulus in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

E modulus of elasticity in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

G_0^* initial G^* in the Taylor series expansion in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

H^0 initial hyperstress in the Taylor series expansion in $\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$

K bulk modulus in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

N coupling number in $\left[\frac{\mathrm{Pa}^2}{\mathrm{Pa}^2}\right]$ or [–]

T^0 initial stress in the Taylor series expansion in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

Third-rank tensor quantities

$\mathbb{D}$ third-rank micro-dilatation theory centro-symmetric tensor in the Taylor series expansion in $\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$

$\mathcal{D}^2 u = u_{i,jk}\hat{e}_i \otimes \hat{e}_j \otimes \hat{e}_k$ third-rank tensor defined as second derivation of displacement vector in $\left[\frac{\mathrm{m}}{\mathrm{m}^2}\right]$ or $\left[\frac{1}{\mathrm{m}}\right]$

D third-rank stiffness tensor $\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$ or [Pa.m]

e third-rank permutation symbol or Levi-Civita tensor [–]

Fifth-rank tensor quantities

$\mathcal{D}^4 u = u_{i,jklm}\hat{e}_i \otimes \hat{e}_j \otimes \hat{e}_k \otimes \hat{e}_l \otimes \hat{e}_m$ fifth-rank tensor defined as fourth derivation of the displacement vector in $\left[\frac{\mathrm{m}}{\mathrm{m}^4}\right]$ or $\left[\frac{1}{\mathrm{m}^3}\right]$

Second-rank tensor quantities

$\bar{U}$ stretch tensor [–]

ϵ infinitesimal engineering strain tensor $\left[\frac{\mathrm{m}}{\mathrm{m}}\right]$ or [–]

$\exp(\nabla u)$ matrix exponent function of gradient displacement [–]

$\mathbf{1}$ second-rank identity tensor or so-called identity matrix [–]

$\mathbb{B}$ micro-dilatation coupling modulus matrix in the Taylor series expansion in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

$\mathcal{D}u = u_{i,j}\hat{e} \otimes \hat{e}_j$ second-rank tensor well known as $\nabla_{\bar{X}} u = \nabla u := (\nabla \otimes u)^T$ in $\left[\frac{\mathrm{m}}{\mathrm{m}}\right]$ or [–]

$\mathcal{E}$ Cauchy–Green strain tensor in [–]

$\nabla \otimes u$ tensorial gradient of displacement vector [–]

$\nabla u := (\nabla \otimes u)^T$ displacement gradient tensor $\left[\frac{\mathrm{m}}{\mathrm{m}}\right]$ or [–]

$\nabla_{\bar{X}} u$ Lagrangian displacement gradient [–] or $\left[\frac{\mathrm{m}}{\mathrm{m}}\right]$

σ stress tensor $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

σ^{CA} Cauchy stress tensor $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

σ^{MD} micro-dilatation stress tensor $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

$\mathbf{F}$ second-rank deformation gradient tensor in [–] or $\left[\frac{\mathrm{m}}{\mathrm{m}}\right]$

Ξ second rank micro-strain tensor [–]

B second-rank micro-dilatation coupling tensor $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

$Q = \cos(x_i, x_j')e_i \otimes e_j$ arbitrary orthogonal transformation second-rank tensor [–]

T stress tensor in the vicinity of small E, Φ and φ in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

Scalar quantities

$\bar{\eta}$ visco-elastic term at pore-scale $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

$\bar{\zeta}$ pore-dependent body force scalar $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

$\bar{p}$ dilatation variable [–]

Λ volumetric matrix fraction at current configuration $\left[\frac{\mathrm{m}^3}{\mathrm{m}^3}\right]$ or [–]

Λ_0 volumetric matrix fraction at reference configuration $\left[\frac{\mathrm{m}^3}{\mathrm{m}^3}\right]$ or [–]

$\mathbb{I}$ micro-dilatation theory centro-symmetric scalar in the Taylor series expansion in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

Ω bulk volume at current configuration [m^3]

Ω_0 bulk volume at reference configuration [m^3]

Ω_{M_0} matrix volume at reference configuration [m^3]

Ω_M matrix volume at current configuration [m^3]

ρ bulk density at current configuration in $\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$

$\rho\dot{Q}$ heat source/sink rate in $\left[\frac{\mathrm{J}}{\mathrm{m}^3\mathrm{s}}\right]$

$\rho\dot{s}$ volumetric entropy rate in $\left[\frac{\mathrm{J}}{\mathrm{m}^3\ \mathrm{s\ K}}\right]$

ρ bulk density with voids in $\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$

$\rho_\ell \ell$ equilibrated body force $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

e free energy per mass in $\left[\frac{\mathrm{J}}{\mathrm{kg}}\right]$

P porosity at current configuration $\left[\frac{\mathrm{m}^3}{\mathrm{m}^3}\right]$ or [–]

P porosity at reference configuration $\left[\frac{\mathrm{m}^3}{\mathrm{m}^3}\right]$ or [–]

Quad(W) quadratic part of total strain energy density extracting from the Taylor series expansion in $\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]$

Θ temperature in [K]

φ gradient of micro-dilatation variable $\left[\frac{1}{\mathrm{m}}\right]$

Di dissipation in $\left[\frac{\mathrm{Pa}}{\mathrm{s}}\right]$

G equilibrated scalar micro-body force in the vicinity of small E, Φ and φ in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

$g = P - S$ equilibrated scalar micro-body force $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

G^* time-independent part of g in the vicinity of small E, Φ and φ in $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

$J = \det(F)$ determinant of deformation gradient tensor [–]

P hydrostatic pressure $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

q heat flux rate in $\left[\frac{\mathrm{J}}{\mathrm{m}^2\ \mathrm{s}}\right]$

S total hydrostatic pressure which differs P due to the independence of dilatation $\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ or [Pa]

$W(E, \Phi, \varphi)$ total energy density of the micro-dilatation theory in $\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]$

W_0 initial total strain energy density in $\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]$

$W(F)$ total strain energy density in $\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]$

$W_{\mathrm{MD}}(E, \Phi, \varphi)$ strain energy density (classical part) of the micro-dilatation theory in $\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]$

$W_{\mathrm{VD}}(E, \Phi, \varphi)$ total void energy density of the micro-dilatation theory in $\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]$