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Stress triaxiality and Lode angle along surfaces of elastoplastic structures

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ABSTRACT

Expressions for the stress triaxiality and the Lode angle along surfaces of elastoplastic structures are established in case of monotonic loading. The stress triaxiality is shown to be governed by the accumulated plastic strain when traction free boundary condition is considered. The exact expressions obtained are generalized to any loading thanks to the proposal of a multiaxiality rule or heuristics whose two parameters are determined from elastic computations of the structure considered: a first one with the elastic properties of the material, a second one quasi-incompressible. The multiaxiality rule proposed can then deal with both plane strain and plane stress conditions. The stress triaxiality at the surface is shown related to the Lode angle. The corresponding expressions are validated on different structures and loadings. Finally, two applications are presented: the enhancement of energetic methods for plasticity post-processing and the enhancement of homogenization localization laws.

1. Introduction

The stress triaxiality defined as the ratio of mean stress or hydrostatic stress divided by von Mises equivalent stress and the Lode angle are a matter of interest in many mechanical fields as soon as the studied phenomena are influenced by the stress state. Ductile damage theories introduced by the early works of McClintock (1968), Rice and Tracey (1969) and then Gurson (1977) exhibit a void growth rate governed by the plastic strain rate but exponentially enhanced by the stress triaxiality (refer to Pineau and Pardoen (2007) for a review). Stress triaxiality is one of the main sensitive quantity for continuous damage and ductile failure (Lemaitre, 1971; Hayhurst and Leckie, 1973; Hult and Broberg, 1974; Hancock and Mackenzie, 1976; Murakami and Ohno, 1978; Beremin, 1981; Krajcinovic and Fonseka, 1981; Lemaitre and Chaboche, 1985; Johnson and Cook, 1985; Rousselier, 1987; Becker et al., 1988; Bernauer et al., 1999; Berdin et al., 2004; Lemaitre and Desmorat, 2005; François et al., 2012; Lemaitre et al., 2009). In recent works (Bao and Wierzbicki, 2004; Xue and Wierzbicki, 2008; Bai and Wierzbicki, 2008), the Lode angle is shown to play a major role on the fracture locus but also on void growth (Nahshon and Hutchinson, 2008) at low stress triaxiality. The stress triaxiality is a matter of interest in surface integrity as compressive residual stresses are sought to improve the fatigue life (Field and Kahles, 1964; Field and Kahles, 1971; Jawahir et al., 2011). It is also a matter of interest in the study of diffusive phenomena in stressed solids, for instance in hydrogen embrittlement (Simpson, 1981; Huez et al., 1998). The solid diffusion is related to the atoms spacing and is obviously made easier in equi-biaxial tension than in uniaxial tension or in compression.

Most of the problems involving these phenomena occur at surfaces and usually require elastoplastic computations when yielding occurs.

It has been shown in a previous work (Desmorat, 2002) that the stress triaxiality at surfaces of structures subjected to monotonic loading is related to the accumulated plastic strain *in plane strain condition*. There is no systematic studies for more general multiaxial states, even at surfaces. Desmorat (2002) work can be extended to a wider range of stress state using different multiaxial constraints, i.e. different assumptions for the multiaxiality of the state of stresses or strains – or mixed quantities – at the surfaces. Such assumptions have mainly been developed in the attempts to extend the fast energetic methods, such as Neuber (1961) and Molski and Glinka (1981) methods, to 3D structural cases (Walker, 1977; Chaudonneret and Culie, 1985; Hoffmann and Seeger, 1985). One of them, giving good results for axisymmetric notched structures, is Hoffmann and Seeger (1985) assumption, that considers a constant strain ratio at the stress concentration point during loading, strain ratio determined from an elastic computation. However, this assumption does not apply to uniaxial stress states. None of the literature assumptions automatically deals with both plane stress and plane strain conditions.

The aim here is to characterize surfaces stress/strain multiaxiality through the values of the stress triaxiality and of the lode

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angle. Lode angle and stress triaxiality are shown to be related on a free surface of an elasto-plastic structure, they are functions of the accumulated plastic strain in monotonic cases. First part of present work concerns the proposal of a novel assumption for the strain multiaxiality at surfaces, called next multiaxiality rule. Expressions for the stress triaxiality established by Desmorat (2002) are then extended to any stress/strain state along surfaces through the consideration of the proposed heuristics. These expressions are assessed on structural examples. Finally, two applications are presented. The first one is the enhancement of energetic methods for plasticity post-processing, the second one the enhancement of homogenization localization laws.

2. Existing stress/strain multiaxiality assumptions

Three dimensional general stress states are usually difficult to handle in closed-form expressions as stress and strain components may evolve independently. This evolution is constrained by the geometry and/or the loading. For plane stress and for plane strain conditions relations in terms of stress or strain components exist and can be used to determine analytically the stress triaxiality (Walker, 1977; Desmorat, 2002). However, in the general case, the multiaxial constraint must be either numerically determined, or – as for fast energetic methods – it is hidden in the general set of equations (Neuber, 1961; Molski and Glinka, 1981; Hoffmann and Seeger, 1985; Moftakhar et al., 1995; Gallerneau, 2000; Chaboche, 2007; Herbland et al., 2007).

2.1. Stress state at surfaces

Obviously, the stress state occurring along surfaces is either uniaxial or biaxial even if the rest of the structure presents a complex tridimensional stress state. In the principal coordinate system (Fig. A.1), the direction associated to principal stress σ_1 is considered normal to the free surface (free surface condition: $\sigma_1 = 0$) and the convention for the principal stresses $\sigma_2 \geqslant \sigma_3$ is adopted.

The free surface condition gives then $\boldsymbol{\sigma} = \mathrm{diag}[0, \sigma_2, \sigma_3]$ so that the three stress invariants are defined respectively as the hydrostatic stress,

$$\sigma_H = \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma}) = \frac{1}{3}(\sigma_2 + \sigma_3) \tag{1}$$

as von Mises equivalent stress,

$$\sigma_{eq} = \sqrt{\frac{3}{2}\boldsymbol{\sigma}' : \boldsymbol{\sigma}'} = \sqrt{\sigma_2^2 + \sigma_3^2 - \sigma_2\sigma_3} \tag{2}$$

and as the third invariant of the deviatoric stress, here made homogeneous to a stress,

$$r = \left[\frac{27}{2}\det(\boldsymbol{\sigma}')\right]^{1/3} = \left[\frac{1}{2}(\sigma_2 - 2\sigma_3)(2\sigma_2 - \sigma_3)(\sigma_2 + \sigma_3)\right]^{1/3} \tag{3}$$

where $\boldsymbol{\sigma}'$ is deviatoric stress tensor,

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \frac{1}{3}\mathrm{tr}\ \boldsymbol{\sigma}\ \mathbf{1} \tag{4}$$

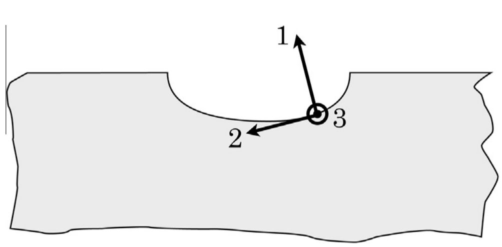

Fig. A.1. Notations for the coordinate system associated to the principal directions of the stress and strain tensors. Direction 1 is normal to the free surface.

The stress triaxiality denoted T_X in present work and the Lode angle Θ are both dimensionless invariants defined respectively with previous stress invariants or components as

$$T_X = \frac{\sigma_H}{\sigma_{eq}} = \frac{1}{3}\frac{\mathrm{tr}\boldsymbol{\sigma}}{\sigma_{eq}} = \frac{\sigma_2 + \sigma_3}{3\sqrt{\sigma_2^2 + \sigma_3^2 - \sigma_2\sigma_3}} \tag{5}$$

$$\cos(3\Theta) = \left(\frac{r}{\sigma_{eq}}\right)^3 = \frac{(\sigma_2 - 2\sigma_3)(2\sigma_2 - \sigma_3)(\sigma_2 + \sigma_3)}{2\left[\sigma_2^2 + \sigma_3^2 - \sigma_2\sigma_3\right]^{3/2}} \tag{6}$$

With these conventions, a stress triaxiality equal to 1/3 corresponds to uniaxial tension and −1/3 to uniaxial compression. Pure shear stress state presents a stress triaxiality equal to zero, in equibiaxial tension the triaxiality remains equal to 2/3 even after yielding. However, the stress triaxiality is in general not constant, even at surfaces, and evolves with respect to the loading.

To describe constraint assumptions at surfaces, it is convenient to define the following stress and strain ratios in terms of the principal stress and strain components at surfaces:

$$\lambda_3 = \frac{\sigma_3}{\sigma_2} \quad \text{and} \quad \phi_1 = \frac{\epsilon_1}{\epsilon_2}, \quad \phi_3 = \frac{\epsilon_3}{\epsilon_2} \tag{7}$$

These ratios are defined with components obtained from an elasto-plastic analysis (lowercase Greek letters).

It is also possible to define similar ratios (still at the surface) but from the principal stresses and strains obtained from a linear elastic analysis of the same structure:

$$\Lambda_3 = \frac{\Sigma_3}{\Sigma_2} \quad \text{and} \quad \Phi_1 = \frac{E_1}{E_2}, \quad \Phi_3 = \frac{E_3}{E_2} \tag{8}$$

Capital notation means then, "components obtained from a linear elastic analysis", $\boldsymbol{\Sigma}$ and $\boldsymbol{E}$ being the stress and strain tensors obtained in elasticity.

2.2. Existing multiaxiality assumptions linking elastic and elasto-plastic quantities

A wide range of assumptions has been proposed in the context of the extension of fast energetic methods to multiaxial stress state at stress concentration points such as notches (Walker, 1977; Chaudonneret and Culie, 1985; Hoffmann and Seeger, 1985; Moftakhar et al., 1995; Singh et al., 1996; Gallerneau, 2000; Knop et al., 2000; Buczynski and Glinka, 2003; Sethuraman and Viswanadha Gupta, 2004; Lim et al., 2005; Chaboche, 2007; Herbland et al., 2007; Ye et al., 2008). They equal elastic and elasto-plastic stress, strain, or mixed ratios, i.e. quantities calculated in elasticity to the same quantities in elasto-plasticity.

The assumption

$$\lambda_3 = \frac{\sigma_3}{\sigma_2} \approx \frac{\Sigma_3}{\Sigma_2} = \Lambda_3 \tag{9}$$

was first introduced by Walker (1977) who assumed a constant stress ratio in combination with Neuber's rule to determinate the stress and the strain state at notch tip of different geometries. It corresponds to a constant stress triaxiality, i.e. a stress triaxiality identical in elasticity and in plasticity,

$$T_X = \frac{1 + \lambda_3}{3\sqrt{1 - \lambda_3 + (\lambda_3)^2}} = \frac{1 + \Lambda_3}{3\sqrt{1 - \Lambda_3 + (\Lambda_3)^2}} = T_X^{\mathrm{elasticity}} \tag{10}$$

As already mentioned, this holds for some special stress states only, as for shear, uniaxial tension and equi-biaxial plane stress tension/compression. However, this does not hold for plane strain conditions and many other intermediate states, even for small plastic strains.