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Generalized domain-independent interaction integral for solving the stress intensity factors of nonhomogeneous materials

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ABSTRACT

The generalized domain-independent interaction integral (DII-integral) is investigated due to its extremely promising application in solving the stress intensity factors (SIFs) of nonhomogeneous materials with complex interfaces. A recent study revealed that the DII-integral, which is domain-independent for material interfaces, can be established by defining an appropriate auxiliary field. Regrettably, the kind of auxiliary field that can be used is not very clear. This work first discusses the conditions that the auxiliary field must satisfy for establishing a DII-integral and provides a framework for designing more applicable auxiliary fields. In this framework, a generalized auxiliary field and the corresponding generalized DII-integral are derived. The generalized auxiliary field contains two free constants and can be expressed as the linear combination of a widely used auxiliary field and a new auxiliary field, which are referred to as the crack face traction-free auxiliary field and the zero mean stress auxiliary field, respectively. The generalized auxiliary field is effective in establishing the DII-integral for purely mechanical loading, and the zero mean stress auxiliary field is also effective in the establishment of the DII-integral for isotropic mismatch strain problems. The generalized DII-integral is the linear function of mode-I and mode-II SIFs and its expression does not involve the free constants in the generalized auxiliary field. Then, a patched extended finite element method (XFEM) is briefly introduced to remove the crack-tip enrichment and instead employ a patched mesh in the modeling. The patched XFEM does not depend on the material constitutive relations and thus has a larger applicable scale than does the traditional XFEM. Finally, the DII-integral combined with the patched XFEM is employed to investigate four representative crack problems. Numerical examples show that the generalized DII-integral exhibits good domain-independence for homogeneous, nonhomogeneous, and discontinuous properties. Although the numerical values of the DII-integral do not remain constant for different values of the free constants due to numerical errors, good numerical precision can be achieved if the free constants do not deviate too much from those used in the crack face traction-free auxiliary field. In addition, the DII-integral using the zero mean stress auxiliary field is demonstrated to be reliable and convenient in solving the SIFs of particulate composites under an initial thermal strain.

1. Introduction

Among the available methods for solving the critical parameters characterizing the asymptotic fields near the crack tip, Rice's *J*-integral (Rice, 1968) has experienced great success because of its path-independence for homogeneous materials. However, the *J*-integral cannot distinguish between contributions due to crack opening and those due to shear in mixed-mode crack problems. To separate mode-I and mode-II stress intensity factors (SIFs), Chen and Shield (1977) developed an interaction integral (*I*-integral) consisting of cross terms in the *J*-integral under the superposition of the actual state and a known auxiliary state. Due to the designability of the auxiliary state, the *I*-integral was subsequently developed for solving the SIFs of two-dimensional (2D) anisotropic solids (Wang et al., 1980) and for bimaterial interface cracks (Nakamura, 1991). Apart from decoupling the SIFs, other researchers (Kfouri, 1986; Cho et al., 1994) demonstrated that the *I*-integral is a powerful tool for extracting the T-stress which is the second term in the eigenfunction expansion of the stress field at the crack tip. The path-independent integral

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including the J-integral and the I-integral has attracted increasing interest, but evaluation of the contour integral in finite element computations is a potential source of inaccuracy (Moran and Shih, 1987). To eliminate the potential inaccuracy, Moran and Shih (1987) proposed to convert the contour integral into an equivalent domain integral. Gosz et al. (1998) and Gosz and Moran (2002) developed equivalent domain integrals of the I-integral to solve the SIFs of three-dimensional (3D) interior and interface cracks.

The I-integral has been demonstrated to be quite effective for solving the SIFs of functionally graded materials (FGMs), which constitute a category of typical nonhomogeneous materials with smooth properties that can reduce the mismatch of material properties and improve the bonding strength and toughness in some structures (Guo and Noda, 2007). Dolbow and Gosz (2002) first developed the I-integral to solve the SIFs of FGMs. In asymptotic expansions of the stress and displacement fields near the crack tip in an infinite homogeneous medium, they adopted the first terms as the auxiliary stress and displacement fields. The auxiliary strain was obtained from the auxiliary stress through the constitutive relations of FGMs. Kim and Paulino (2003a, 2003b, 2003c, 2005) gave a series of representative works on the I-integral for solving the SIFs and the T-stress of FGMs. They proposed three alternative definitions of the auxiliary fields, i.e., non-equilibrium, incompatibility, and constant-constitutive-tensor formulations, and showed that these three formulations are effective for establishing the I-integral for both isotropic and orthotropic FGMs. Following their idea, Walters et al. (2006) developed three auxiliary fields for establishing the I-integral for 3D FGMs. Amit and Kim (2008) demonstrated the effectiveness of these three auxiliary fields in establishing the I-integral for solving the SIFs and T-stress of FGMs under thermal loading. According to the definitions given by Kim and Paulino (2005), Rao and Kuna (2008a, 2008b) proposed three auxiliary fields for establishing the I-integral for functionally graded piezoelectric and magneto-electro-elastic materials. Recently, the I-integral was developed for solving the intensity factors of stationary dynamic cracks in piezoelectric and magneto-electro-elastic solids (Bui and Zhang, 2012, 2013; Sharma et al., 2013). Liu et al. (2013, 2014) derived the I-integral for functionally graded piezoelectric materials under transient mechanical and thermal loading. These research works show that the I-integral is a reliable method for nonhomogeneous materials with continuous properties.

Actually, interfaces in nonhomogeneous materials generally act as defect sources due to the deformation mismatch, and so materials with complex interfaces are more prone to fracture failure than are materials with continuous properties. Conventional I-integrals are rarely used for materials with complex interfaces because both the contour integrals and their equivalent domain integrals must be kept away from the interfaces. Fortunately, Yu et al. (2009, 2010, 2012a, 2012b) recently found that the I-integral has the attractive feature of being domain-independent for material interfaces, when using an appropriate auxiliary field; therefore, the I-integral is an extremely promising method in the fracture analysis of materials with complex interfaces. The I-integral with this feature is referred to as the domain-independent I-integral (DII-integral) and its superiority is shown in the crack problems of bi-materials, FGMs, and fiber-reinforced composites (Pathak et al., 2012; Bhattacharya et al., 2013; Cahill et al., 2014). However, the previous studies did not mention the reason why this auxiliary field can be used to establish the DII-integral. Additionally, Guo et al. (2012) found that the I-integral loses the domain-independence feature for interfaces with thermal mismatch strain.

To explore the kind of auxiliary field valid for the DII-integral and to find a method to overcome the difficulty caused by thermal mismatch strain, the present paper first discusses the continuity conditions that the auxiliary field must satisfy to establish a DII-integral. Accordingly, we provide a framework for designing more applicable auxiliary fields. In this framework, we derive a generalized auxiliary field that contains two free constants. By assigning appropriate values to these constants, the generalized auxiliary field reduces to two special auxiliary fields. One is the crack face traction-free auxiliary field, which has been widely used in previous studies (Dolbow and Gosz, 2002; Kim and Paulino, 2005; Yu et al., 2009), and the other is the zero mean stress auxiliary field, which has a mean auxiliary stress of zero. Compared with the DII-integral in previous studies (Yu et al., 2009; Pathak et al., 2012), the generalized DII-integral developed in this paper needs to compute a crack face integral for a traction-free crack, but it is not necessary to apply the analytical solutions of the crack-tip elastic fields as the auxiliary field. Especially, it is found that the zero mean stress auxiliary field can be used to establish the DII-integral for interfaces with isotropic mismatch strain. A local mesh replacement method based on the extended finite element method (XFEM) is introduced briefly. In this method, only the enrichments for the elements that are cut completely by an interface and completely by a crack are used in the approximation of the displacement, whereas the enrichments for the elements partially cut by a crack are removed. Instead, a refined mesh containing singular elements at the crack tip is patched on the crack-tip region to achieve high numerical precision. Since a patched mesh is used in the modeling, this method is referred to as the patched XFEM. The patched XFEM does not depend on the material constitutive relations and thus has a larger applicable scale than the traditional XFEM. Finally, we present four representative numerical examples to verify the validity of the generalized DII-integral and check the influence of different auxiliary fields on the numerical results.

2. DII-integral

The DII-integral is an effective and convenient method for evaluating the SIFs and the T-stress of materials with complex interfaces, because it is not necessary to keep the integral domain away from interfaces. The present study aims to establish a generalized DII-integral for solving the SIFs of a 2D isotropic elastic solid.

2.1. Definition of the I-integral

The J-integral (Rice, 1968) is defined as follows:

$$J = \lim_{\Gamma_\varepsilon \to 0} \int_{\Gamma_\varepsilon} (W\delta_{1i} - \sigma_{ij}u_{j,1})n_i d\Gamma \tag{1}$$

where $W = \sigma_{ij}\varepsilon_{ij}/2$ is the strain energy density, u_i, σ_{ij}, and ε_{ij} are respectively the displacement, stress, and strain tensors, δ_{ij} is the Kronecker delta, and n_i is the unit outward normal vector to a contour, as shown in Fig. 1. In the present paper, the subscripts i, j, k, l take on the values 1 and 2, and the repetition of an index in a term

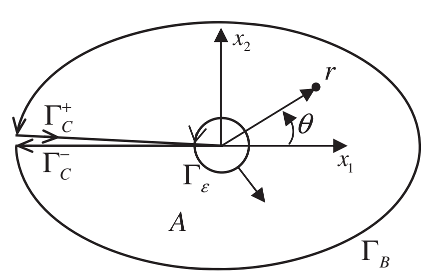

Fig. 1. Schematic diagram of crack-tip contours.

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