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Optimal cold bending of laminated glass

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ABSTRACT

Cold-bending of laminated glass panels, by forcing their contact with a constraining frame, is a promising technique for free-form glazed surfaces. Their static state varies in time due to the viscosity of the polymeric interlayer, which causes the decay of the shear-coupling of the constituent glass plies. The direct problem consists in calculating the spatial and temporal evolution of stress after cold-bending. Considering an equivalent secant elastic shear-modulus for the interlayer to account for its viscoelasticity, various conditions for cylindrical deformations are analyzed in detail. A "conjugate-beam analogy" is proposed for the inverse problem, i.e., to determine the deformed shape that, at a prescribed time, provides the desired state of stress. Remarkably, the simplest constant-curvature deformation, often used for cold bending, produces high shear stress concentrations in the interlayer with consequent risks of delamination. For the same sag, better linear or cubic distribution of shear stress are attained with slightly different deformations, compatibly with glass strength. Among the considered cases, the optimal configuration is sinusoidal, because it provides the smoothest distribution of shear stress with inappreciable geometric differences with respect to the circular shape.

1. Introduction

Curved glass is a powerful tool of aesthetic design, and its use is steadily increasing in modern architecture. There are two main categories of production processes: hot bending and cold bending. Hot bent glass is obtained by heating sheets of glass until they reach the softening point (glass transition temperature) and curving them into the desired shape against a negative form. Cold bending is a recent fabrication process that is widely developing because it allows for the construction, at relatively low cost, of curved free-form glazed surfaces with no need of negative moulds. In general, the cold bent surface is a single-curvature developable surface. Cold bending into a double curved shape is also possible (Beer, 2013; Galuppi et al., 2014), but since this produces high membrane stress, single-curvature bending remains the most used technique, also because recent advances in theoretical algorithms allow for the discretization of any surface using single curvature panels only. Therefore, large double-curvature glazing of any form can be approximated by cylindrically bent panels (Pottmann et al., 2008; Eigensatz et al., 2010).

In cold bending, flat glass panels are brought to the desired geometry by external contact forces, constraining the curved glass unit in the desired shape. The most common technique[1] consists in curving glass at the construction site, holding it in place with clamps or adhesives against an underlying frame. Laminated glass is particularly adapt for cold bending. This is a sandwich structure composed by two or more glass plies bonded together by thin polymeric interlayers with a process at high temperature and pressure in autoclave. The limited shear coupling of the glass plies through the interlayer (Behr et al., 1993; Hooper, 1973) reduces the overall stiffness of the panel, increasing the maximum attainable curvature through cold bending compatibly with the material strength. As pointed out by Norville et al. (1998), in general the bending stiffness of laminated glass is intermediate between the *layered limit* (free-sliding glass plies) and the *monolithic limit* (shear-rigid interlayer). Since stress and strain in the monolithic limit are much lower than in the layered limit, appropriate consideration of the shear coupling offered by the interlayer is important to achieve an economical design. The problem has been considered by many authors, one of the most recent contribution being the careful finite element analysis by Ivanov (2006) that includes a list of the most relevant literature.

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[1] Another technique consists in laminating a package while being constrained in the desired shape, so that after lamination it is the bond of the interlayer that keeps the assembly in the curved state. This procedure, usually denoted *cold lamination bending* (Kassnel-Henneberg, 2011; de Vericourt; Fildhulth and Knippers, 2011), is not considered here.

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Due to the viscosity of the polymer, as remarked by Belis et al. (2007), the cold-bent laminated glass element exhibits a stress relaxation from the instant in which it is completely positioned in its curved frame. These effects must be precisely predicted in order to evaluate the temporal variation of the state of stress.

The shear coupling provided by the polymeric interlayer depends upon its viscoelastic response, which is highly time- and temperature-dependent (Louter et al., 2010; Bennison et al., 2005; Froli and Lani, 2011; Barredo et al., 2011; Galuppi and Royer-Carfagni, 2012). In the design practice it is customary to rely upon approximate solutions, the most common of which considers the polymer as a linear elastic material, characterized by a proper secant shear modulus, calibrated according to temperature and characteristic duration of the design actions (Bennison and Stelzer, 2009). Such an approximation, usually referred to as the *secant stiffness* method or *quasi elastic* approximation, is equivalent to neglecting the memory effect of the viscoelastic material. This in general provides estimates on the side of safety in the case of monotone loading histories (Galuppi and Royer-Carfagni, 2012), even if there may be loading–unloading paths for which the results are not conservative, as demonstrated in Galuppi and Royer-Carfagni (2013).

Here, the single-curvature cold-bending of a laminated glass panel is analyzed. Due to the hypothesis of cylindrical deformation, the problem is tackled by using sandwich beam theory, developing a method originally proposed by Newmark et al. (1951) to evaluate the relationship between the prescribed cold-bent shape and the spatial and temporal evolution of the state of stress in both glass and polymeric interlayer. Stress relaxation is calculated by adopting the secant stiffness approximation. The model allows to solve the direct problem, i.e., to find the state of stress for any given assigned deformation of the laminated glass beam.

The most used shape for cold bending is the constant-curvature shape. However, it will be analytically proved that such a configuration is associated with shear stress concentrations in the polymeric interlayer, possibly producing delamination as sometimes observed in the practice. The higher the shear stiffness of the interlayer, the more critical is its state of stress. In the limit case of stiff interlayers (monolithic limit), the shear stress becomes singular because concentrated forces at the extremities are necessary to guarantee equilibrium.

A “conjugate-beam analogy” is then proposed to solve the inverse problem, i.e., to determine the cold-bending shape associated with an assigned shear stress distribution in the interlayer at a prescribed time of the history. In fact, it is shown that such shape coincides with the deformation of a conjugate beam under a fictitious load and appropriate boundary conditions, which are determined by the form of the desired shear stress. Various types of shear stress distributions are analyzed in detail. Remarkably, the simplest constant-curvature shape is the one that produces the highest shear stress in the interlayer. Slightly modifying the configuration due to cold bending, better linear or cubic distributions of shear stress can be obtained. Among all the considered cases, the optimal configuration is the sinusoidal deformation, associated with a cosine distribution of shear stress in the interlayer that allows to obtain the maximum sag of the laminated package, compatibly with the strength of glass and polymer. For standard geometric parameters, the difference between the sinusoidal and the circular shapes cannot be appreciated with the naked eye, and consequently the aesthetics is not compromised. Indeed, so small differences in the constrained deformations can provide so noteworthy advantages.

2. Cold bending: mathematical model

Consider a laminated glass beam of length L and width b, composed by two glass layers of thickness h_1 and h_2 and Young's modulus E, bonded by a thin polymeric interlayer of thickness h with time-dependent shear modulus $G(t)$. Introduce a right-handed orthogonal reference frame (x, y), with x parallel to the beam axis and y directed upwards, as indicated in Fig. 1(a). Perfect bonding between glass and polymeric interlayer is assumed and, under the hypothesis that strains are small and rotations moderate, the prescribed vertical displacement $v(x)$, assumed positive if in the same direction of increasing y, is the same for all the three layers. The *cold bending* process consists in forcing the beam to assume a curved shape. From a practical point of view, the laminated glass plate is glued or clamped along its border onto a negative curved frame, so to assume a cylindrical deformation as schematically represented in Fig. 1(b), where the bond thickness is supposed to be negligible. In the assumed beam model, this is equivalent to assigning the vertical displacement $v(x)$.

The viscoelasticity of the interlayer induces the relaxation of the shear coupling of the glass ply, so that the macroscopic bending stiffness of the beam varies with time. Consequently, the bending moment $M(x,t)$ is time-dependent, as well as the constraint reaction forces per unit length $p(x,t)$. Denoting, here and further, with $'$ derivative with respect to x, and assuming that $M(x,t) > 0$ when $v''(x) > 0$, equilibrium of an elementary portion of the beam gives

$$p(x,t) = -M''(x,t), \tag{2.1}$$

with $p(x,t)$ positive if directed downwards.

2.1. Viscoelasticity of the interlayer and secant-stiffness approximation

The viscoelastic properties of the interlayer[2] can be interpreted through the Maxwell–Wiechert model (see Wiechert, 1893), according to which, under constant shear-strain, the shear modulus of the viscoelastic material decays with time according to the Prony series

$$G(t) = G_\infty + \sum_{k=1}^{N} G_k e^{-t/\theta_k} = G_0 - \sum_{k=1}^{N} G_k (1 - e^{-t/\theta_k}), \tag{2.2}$$

where G_∞ is the long-term shear modulus (corresponding to the totally relaxed material), whereas the terms G_k and $\theta_k, k = 1 \dots N$, are respectively the relaxation shear moduli and the relaxation times associated with the kth Maxwell element. The instantaneous shear modulus G_0 is thus given by $G_\infty + \sum_{k=1}^{N} G_k$. Temperature dependence may be taken into account by using the Williams–Landel–Ferry model (Williams et al., 1955). Parameters that define the Prony series are seldom furnished by the producer (Bennison and Stelzer, 2009), but they can be directly measured.

In general, the stress in the interlayer depends upon the whole strain history (Galuppi and Royer-Carfagni, 2012), but a very common practical approach consists in adopting the *secant stiffness* approximation, according to which the polymer behaves as a linear elastic material, whose elastic shear modulus depends upon temperature and characteristic duration of the design actions. As discussed by Galuppi and Royer-Carfagni (2012) and Galuppi and Royer-Carfagni (2013), this is equivalent to assume that the stress $\tau(x,t)$ in the interlayer is a linear function of the shear strain $\gamma(x,t)$ according to an expression of the form

$$\tau(x,t) = G(t)\gamma(x,t), \tag{2.3}$$

where $G(t)$ is the shear modulus. The use of such an approximation is particularly effective because there are several practical methods

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[2] There are essentially three main commercial polymeric films: Polyvinyl Butyral (PVB), Ethylene Vinyl Acetate (EVA), and ionoplast (Bennison et al., 2001). For cold bending, it is convenient to use a soft interlayer, to diminish the shear coupling of the glass plies and thus reduce the bending stiffness of laminated glass. For this reason, one of the best choices is certainly PVB, a polyvinyl acetate with addition of softeners that provide plasticity and toughness, enhancing adhesion-strength and increasing glass transition temperature up to 20–25 °C.