International Journal of Solids and Structures 66 (2015) 1–19

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Analytical mean Green operators/Eshelby tensors for patterns of coaxial finite long or flat cylinders in isotropic matrices

P. Franciosi [a,*], S. Barboura [b], Y. Charles [b]

[a] CNRS, LSPM UPR3407, Université Paris 13, Sorbonne Paris Cité, 93430 Villetaneuse, France
[b] Université Paris 13, Sorbonne Paris Cité, LSPM, CNRS UPR3407, 93430 Villetaneuse, France

ARTICLE INFO

Article history:
Received 22 July 2014
Received in revised form 16 February 2015
Available online 13 April 2015

Keywords:
Green operator
Eshelby tensor
Radon transform
Finite cylindrical inclusions
Coaxial cylinders
Fragmented fibers
Aligned platelets
Inclusion patterns

ABSTRACT

Recently, an analytical solution has been provided for mean and axial Green operators (GO) of circular cylindrical inclusions with finite length in 3D isotropic (elastic-like and dielectric-like) materials, from long rods to truly penny-shaped ones. This result provides a situation for examining the commonly used "shape spheroidal approximation" of same aspect ratio as well as possibly more appropriate alternatives. For cases when such a spheroidal approximation would remain needed, we propose and compare an "operator spheroidal approximation" in terms of best matching GO, which is exactly operator-identical for dielectric-like properties and nearly identical for elastic-like ones. Next, a simple analytical solution is explicated for the mean isotropic GO of all patterns of coaxial circular long or flat cylinders with same radii, obtained from solving the coaxial cylinder pair interaction problem. These patterns can be seen as either fragmented cylinders or as cylinder alignments. The obtained GO solution form is further shown to formally hold for similar patterns of domains having more general shapes. The established general property of such patterns is that the interaction part of their mean GO can be expressed as a linear combination of the mean GO of all the pattern sub-domains. The mean GO solution for one such pattern is further proved to be valid as well for the domains – of the double inclusion type – obtained in connecting the pattern elements by their outer envelop of zero thickness, thanks to the GO blindness to connectivity differences. This GO equivalency is finally used to define a global operator spheroidal approximation in terms of best matching GO for any such coaxial cylinder pattern. Effective properties for an isotropic matrix reinforced by congruent cylinders or patterns are estimated within a mean field approximation context.

1. Introduction

Since the pioneering work of Eshelby (1957), developing the applications of ellipsoidal symmetry properties to solving complex mechanical or physical problems has been the goal of a tremendous amount of works. Among milestones of these capability developments one can cite the solving or setting of the ellipsoidal inclusion-pair problem (Berveiller et al., 1987), of the double and multiple ellipsoidal inclusion model (Hori and Nemat-Nasser 1993; Nemat-Nasser, 2000), or of the ellipsoidal symmetry assumption for the spatial distribution of inclusions (Willis, 1977; Ponte Castaneda and Willis, 1995) and of inclusion patterns (Bornert et al., 1996; Buryachenko, 2001).

Aside of the world of ellipsoidal symmetry, an important literature has also been devoted to the calculation of Green operators and thus of Eshelby tensors related to finite domains of various shapes. Indeed, when a mean (volume averaged) Green operator or Eshelby tensor for the considered shape is available (in manageable form), it is in many circumstances preferable to the operator of any "best ellipsoidal approximation", unless such an approximation remains necessary, for example for mathematical purposes. However, apart of the infinite laminate and fiber cases, comparatively much less work has been devoted to finding analytical solutions for other inclusion shapes or patterns, the reason being that solutions, which are no more uniform then, are hardly analytical even in terms of mean values. Polyhedral inclusions which are very frequent in the world of real materials have been for example examined from different viewpoints and contexts in Rodin (1996), Kawashita and Nozaki (2001), Nozaki and Taya (2001), Franciosi and Lormand (2004) and Franciosi (2005). More "exotic" shapes have also been examined with the hope to find other geometries than ellipsoidal possessing interesting properties, as triangular or pentagonal star-shaped inclusions (Mura et al.,

* Corresponding author. Tel.: +33 1 49 40 35 02.
E-mail address: patrick.franciosi@univ-paris13.fr (P. Franciosi).

http://dx.doi.org/10.1016/j.ijsolstr.2015.03.027
0020-7683/

1994, 1995; Mura, 1997) or doughnut-like inclusions (Onaka, 2003, 2005; Onaka et al., 2001, 2003) for examples. Situations involving inclusions of general shapes remain more generally solved with the help of numerical methods, as for example in Nakasone et al. (2000) and Eroshkin and Tsukrov (2005) among others.

A recent review of works on inclusions has been published by Zhou et al. (2013) with many references, but to their and our knowledge, no reported works addressed the case of cylindrical inclusions with finite length in a 3D space. This important missing case in the family of domain shape types has motivated the recently published work in Franciosi (2014), providing an analytical solution for the mean and axial Green operators and Eshelby tensors of finite circular cylinders in media with isotropic properties, from using the Radon/inverse Radon transform (RT/IRT) method. This method which was successfully applied previously to solve analytically some particular pattern or architecture cases (Franciosi, 2010; Franciosi and El Omri, 2011) and to retrieve classical solutions as for the ellipsoid pair (Franciosi and Lormand, 2004), has yield a quite simple solution for finite cylinders.

The present extension to pairs and patterns of finite coaxial cylinders was first motivated by the fact that, as stated by Eroshkin and Tsukrov (2005), in the three-dimensional elasticity, the "menu" of available solutions (for inclusions of non ellipsoidal shapes to be used in micromechanical "inclusion problems") is still very limited. Although this would be enough to make any new solution welcome, regardless of the interest one can presume it to carry, the work also addresses a few points that have been encountered meanwhile, the validity and potential interest of which overpass the cylinder case.

This finite cylindrical shape type does possess some original characteristics. In general, the shapes of the parts of an inclusion have nothing or little to compare with the shape of the plain inclusion they are parts of (ellipsoid parts for example). For general polyhedral inclusions, partitions into polyhedral parts are possible but even in the best cases, partitions into parts of the same "family" as the whole domain can hardly be achieved.[1] In contrast, if a finite cylinder, whether its cross section be of circular, of polygonal or of general shape, is plane-cut one or several times normally to its axis, the obtained pattern of two or more inclusion parts is still a set of inclusions of the same type (smaller cylinders with same cross section). Finite cylinders with circular cross section are the simplest shape of this finite cylinder family, owing to the additional axial symmetry. Cutting finite cylinders into such pieces transforms them into coaxial cylinder patterns. As far as all the elements are kept at their place, all remaining in contact, the difference between the plain cylinder and the pattern is only the connectivity that is changing from simple to multiple. Several other coaxial patterns can be obtained in removing one or several of the elements such that the remaining elements are no more at contact.

Such coaxial cylinder patterns, with distant pieces or at contact, can also be seen as "fragmented cylinders". This "fragmented cylinder", is a particular case of a general fragmented inclusion in the same way, say a pattern made by removing one or several "slices" along a plain inclusion by transversal (not necessarily parallel) planar cuts as exemplified in Fig. 1, for (a) general, (b) cylindrical and (c) spheroidal domains. When the cross section is circular, a strikingly simple analytical solution is presented, from the inclusion pair to any general inclusion number, for the mean shape function as well as for the related mean Green operator and Eshelby tensor in isotropic (elastic or dielectric) media. This is obtained from solving the pair interaction problem between two coaxial cylinders of same radius. This solution for cylinder pairs (Fig. 1(b)) and patterns is here introduced in starting from the solution for the general domain of Fig. 1a, of which ellipsoids and spheroids (Fig. 1(c)) also are a particular case with additional known characteristics.

In order to avoid possible misleading about the image of a "fragmented cylinder" given to the inclusion patterns here of concern, one can also conversely see them – and more conveniently for flat shapes – as a cylinder alignment or pile up, such that the here discussed patterns range from cut long fibers to piled penny shaped ones. And since finite cylinders have not been much examined so far, the here addressed properties or characteristics of this type of inclusions are kinds of primary ones.

The first point we examine is how and how much the newly available mean Green operator solution for the finite cylinder differs from the commonly used approximation by the operator of the spheroid of same aspect ratio, which is a simple and rough shape-fitting approximation. Yet, this mean Green operator is the most important inclusion (and inclusion spatial distribution) characteristic, prior to the shape itself from which it results. It is directly involved in mean interior stress and strain field calculations and in effective property estimates for materials with such a shape as domain or symmetry characteristic. For these reasons, the accuracy or relevancy of its approximations deserves attention. Even for non ellipsoidal shapes or symmetries of which the mean operator is known, circumstances needing a spheroidal approximation still occur, as will be exemplified. For finite circular cylinders, the knowledge of the mean Green operator allows to examining alternative spheroidal approximations to the commonly used shape approximation. We here examine the one based on the condition of a Green operator matching at the best the cylinder mean operator. The differences between the cylinder mean Green operator and these two (shape and operator) spheroidal approximations are discussed in Section 2 after a brief recall of the RT/IRT approach and of the finite cylinder solution. It is shown that for dielectric-like isotropic properties, this best matching operator approximation is operator-identical to the cylinder one while for elastic-like isotropic properties it can only be close or nearly identical.

The second examined problem is the pair interaction problem between two finite cylinders. In contrast with the out of reach analytical solution for a general pair, a strikingly simple solution is obtained for the coaxial cylinder pair of same radii, and from which all the pattern cases of n coaxial cylinders with same radii can be simply solved as well, recurrently. This is done in Section 3, formally generalizing the pattern type to more general inclusion assemblages made by removing one or several "slices" from a simply connected domain or by aligning several domains as will be shown. In deriving this solution, it is established that the interaction part in the mean Green operator of all such patterns can be expressed as a linear combination of the mean Green operators of all the pattern sub-domains. This property provides an alternative to the direct calculation of interaction operators between inclusions.

The third addressed property in Section 4 is the (known but not used) one that all the domains which only differ by their connectivity characteristics share the same mean Green operator. This property here allows to showing that the obtained operator for a coaxial cylinder pattern also holds for the domain obtained in connecting the pattern elements by their outer envelop of zero thickness, what constitutes the interfacial domain of a particular type of so-called "double inclusions" (Hori and Nemat-Nasser, 1993). From this equivalency, it is shown that the operator spheroidal approximation proposed for finite cylinders can also be defined globally for such coaxial cylinder patterns, while a global shape approximation would not make sense for these patterns. Situations when a global spheroidal approximation can be of interest for patterns are exemplified as well.

[1] The more unlikely when the more restrictive is the family definition (same number of faces, of same polygonal faces, of same face shapes, regular or not polyhedra etc).