ARTICLE IN PRESS

International Journal of Solids and Structures xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

An efficient method for solving three-dimensional fretting contact problems involving multilayered or functionally graded materials

Zhanjiang Wang [a,*], Chenjiao Yu [b], Qian Wang [b,a]

[a] State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqing 400030, China
[b] Department of Mechanical Engineering, Northwestern University, Evanston, IL 60208, USA

ARTICLE INFO

Article history:
Received 27 September 2014
Received in revised form 6 March 2015
Available online xxxx

Keywords:
Fretting
Multilayered materials
Functionally graded coatings
Frequency response functions
Material dissimilarity

ABSTRACT

Three-dimensional fretting contacts involving multilayered or functionally graded materials are commonly seen in mechanical systems. The analyses of surface fatigue and contact failure require the knowledge of pressure, shear tractions, and stresses. This paper presents a novel method for analyzing the fretting contacts of these materials. The frictional contact equations are divided into two portions, one containing the unknown contact pressure and the other the shear tractions, solved by using the conjugate gradient method with boundary conditions enforced during the iteration. Displacements and stresses caused by the contact pressure and shear tractions are calculated through the use of the influence coefficients and by means of the fast Fourier transform. The influence coefficients are obtained from the analytical frequency response functions derived by the authors, which are the frequency-domain responses of a multilayered surface system to a unit concentrated normal or tangential force. Functionally graded coatings are modeled with multiple sufficiently thin layers; and the minimum number needed to simulate a functionally graded material is numerically determined. This modeling approach is applied to simulate the fretting contact involving multilayered materials and functionally graded coatings and to unfold the dependence of the tangential load–displacement relationship on the degree of material dissimilarity.

1. Introduction

Functionally graded materials (FGMs), or functionally graded coatings (FGCs), where the mechanical properties vary beneath the surface, are widely used for components in aircraft and aerospace systems, computer and electronic devices, and mechanical and nuclear equipment. The materials with gradient properties can be made more resistant to fatigue and fracture than traditional homogeneous materials (Suresh, 2001). Contact analyses of FGMs are essential to provide the stress and deformation information needed for the understanding of surface and subsurface fatigue damage.

For two-dimensional problems, Giannakopoulos and Pallot (2000) investigated the contact between a rigid cylinder and a FGM with a constant Poisson ratio but varying elastic modulus in the depth following a power law. Guler and Erdogan (2004) studied the sliding contact problem of a graded coating bonded to a homogeneous substrate. However, the coupling between pressure and shear was not considered in the above researches. The frictionless contact involving FGMs with arbitrarily varying elastic modulus was investigated by Ke and Wang (2006) using a multilayered surface structure, where the shear modulus was varied linearly in each layer while the continuity at the interfaces was maintained. Further studies on other types of contacts involving graded coatings have also been reported, such as an adhesive contact (Chidlow et al., 2013), a receding contact (El-Borgi et al., 2006), and a partial slip contact (Chen and Chen, 2013).

For three-dimensional problems, Giannakopoulos and Suresh (1997a) derived closed-form solutions for materials with several special Poisson's ratios for the displacements and stresses produced by a concentrated normal force; however, Young's modulus along the depth only followed a power or an exponential law. The normal indentations of FGMs by axisymmetric inventors, including a flat circular punch, sphere, and circular cone, were further investigated (Giannakopoulos and Suresh, 1997b). More complex contact problems involving FGMs, such as those modeled by the extended Johnson–Kendall–Roberts (JKR) and Derjaguin–Muller–Toporov (DMT) adhesive contact formulations (Chen et al., 2009; Jin et al., 2013) and that involving a receding phenomenon (Rhimi et al., 2009), were also investigated.

* Corresponding author. Tel.: +86 18502376952.
E-mail address: wangzhanjiang001@gmail.com (Z. Wang).

http://dx.doi.org/10.1016/j.ijsolstr.2015.04.010
0020-7683/

Nomenclature

a Hertzian contact radius, mm
$A^{(k)}$, $\overline{A}^{(k)}$, $B^{(k)}$, $\overline{B}^{(k)}$, $C^{(k)}$, $\overline{C}^{(k)}$ unknown coefficients in Papkovich–Neuber potentials in the frequency domain in layer k
$C_p^{\bar{u}_i}$ influence coefficients relating pressure to surface displacements, mm/MPa
$C_p^{\sigma_{ij}^{(k)}}$ influence coefficient relating pressure to stresses in layer k
$C_{q_x}^{\bar{u}_i}$, $C_{q_y}^{\bar{u}_i}$ influence coefficients relating shear tractions to surface displacements, mm/MPa
$C_{q_x}^{\sigma_{ij}^{(k)}}$, $C_{q_y}^{\sigma_{ij}^{(k)}}$ influence coefficients relating shear tractions to stresses in layer k
E_k Young's modulus of layer k, MPa
E_0 Young's modulus at surface, MPa
E_s Young's modulus of the sphere and substrate, MPa
G_k shear modulus of layer k, MPa
F_x, F_y applied tangential load along the x, y direction, N
g surface gap, mm
h_0 initial separation between two surfaces, mm
h_k thickness of layer k, mm
I_c set of all grid nodes in the contact region
I_{slip} set of all grid nodes in the slip zone
I_{stick} set of all grid nodes in the stick zone
J_2 the second invariant of the stress deviator tensor, MPa^2
L total number of layers
m, n Fourier-transformed frequency variables with respect to x and y
M, N number of discrete grid points in the x and y directions
p pressure, MPa
p_h maximum Hertzian pressure, MPa
q_x, q_y shear tractions parallel to the x, y direction, MPa
R radius of the sphere, mm
s_x, s_y relative slip distance parallel to the x, y direction, mm
TOL tolerance
$u_i^{(k)}$ elastic displacement of layer k, mm
$\bar{u}_x$, $\bar{u}_y$, $\bar{u}_z$ surface elastic displacements in three directions, mm
W applied normal load, N
x, y, z_k Cartesian coordinates in the spatial domain, mm
Y shape function
α distance of a node (m, n) to the origin in the frequency domain
δ_x, δ_y, δ_z rigid displacement parallel to the x, y, and z direction, mm
δ_{ij} Kronecker delta
Δ_x, Δ_y, Δ_z grid size in the x, y, and z direction, mm
ΔE energy dissipated per cycle, N·mm
μ_f friction coefficient
ν_k Poisson's ratio of layer k
$\sigma_{ij}^{(k)}$ stress of layer k, MPa
σ_{von} von Mises stress ($\sqrt{3J_2}$), MPa
φ and $\psi(\psi_1, \psi_2, \psi_3)$ Papkovich–Neuber potentials

Special marks
$\approx$ or FT_{xy} double continuous Fourier transform about x and y
^ discrete Fourier transform
IFFT inverse fast Fourier transform

Superscripts or subscripts
$k = 1, \ldots, L$ layer number
m derivative with respect to m

On the other hand, FGMs can be simulated with surface structures of sufficiently thin layers, where the layers of constant elastic properties are bonded together perfectly. The advantage of this method is that the shear modulus or Young's modulus along the depth can follow any law, not limited to lineal, power, or exponential laws that offer mathematical conveniences. This approach requires the basic solution to the contact problem involving a multilayered surface structure. For a single-layered surface structure, the analytical elastic solutions for the contact stresses and displacements were derived for problems involving axisymmetric normal loading (Burmister, 1945) and arbitrary normal loading (Chen, 1971; Chen and Engel, 1972). By using the Papkovich–Neuber potentials and double Fourier transform, O'Sullivan and King (1988) derived the analytical solutions of the displacements and stresses in the frequency domain. Then, the normal contact problem was solved by using the least squares iteration approach. When the loads are unit concentrated normal and tangential forces, the elastic solutions in the frequency domain are called the frequency response functions (FRFs). Following the same method of O'Sullivan and King (1988), the analytical solutions for bi-layered substrate were derived and then applied to solve the contact problems involving rough surfaces (Cai and Bhushan, 2005; Yu et al., 2013). Furthermore, the analytical frequency response functions for a multilayered surface system, defined by recurrence relationships, were derived by Yu et al. (2014). Using the derived fundamental solutions, several elastohydrodynamic lubrication problems involving multilayered materials were successfully solved (Wang et al., 2015).

When the mating materials have different elastic properties, the contact pressure and shear tractions are no longer independent. For two-dimensional problems, Spence (1968, 1973) first obtained several solutions for contact problems subjected to the coupled normal and tangential loads. Nowell et al. (1988) explored the shear tractions occurred in a cyclic contact loading process. Nowell and Hills (1988) analyzed the contact involving an elastic layer resting on a rigid frictionless substrate. Chen and Chen (2013) studied the problem of a graded coating bound onto a rigid substrate indented by a rigid punch. Ke and Wang (2010) developed a linear multilayer-material model for analyzing the fretting contact of FGMs. Liu et al. (2012) used a similar method to study torsional fretting. The finite element method (FEM), as a full numerical method, has been widely used for simulating fretting contacts (McColl et al., 2004; Ghosh et al., 2013). It is also applied to study crack problems (Ghosh et al., 2015). However, the FEM faces a discretization problem. Most of the current studies used relative rough girds, especially for the three-dimensional contact problem, due to the limitation of computer processors and memories. As a result, the coupling between the pressure and shear tractions are largely ignored (Leonard et al., 2011).

Recently, the conjugate gradient method (CGM) was employed (Chen and Wang, 2008; Gallego et al., 2010; Wang et al., 2010) to solve the contact problems characterized with interactions between pressure and shear tractions. This method can deal with the three-dimensional contact problems with complex surface geometry and roughness. Using the CGM, a wide range of frictional contact problems were well solved with satisfactory numerical efficiency, among them are the contacts of single layered surface structures (Wang et al., 2010, 2011b), rough surfaces (Chen and Wang, 2009), elasto-plastic materials (Wang et al., 2013b), and inhomogeneities materials (Wang et al., 2013a), and that under more complex loads involving a tangential force and a twisting moment (Wang et al., 2011a).

Please cite this article in press as: Wang, Z., et al. An efficient method for solving three-dimensional fretting contact problems involving multilayered or functionally graded materials. Int. J. Solids Struct. (2015), http://dx.doi.org/10.1016/j.ijsolstr.2015.04.010