International Journal of Solids and Structures 66 (2015) 77–87

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

On the parameters which govern the symmetric snap-through buckling behavior of an initially curved microbeam

X. Chen, S.A. Meguid *

Mechanics and Aerospace Design Laboratory, Department of Mechanical and Industrial Engineering, University of Toronto, 5 King's College Road, Toronto, Ontario M5S 3G8, Canada

ARTICLE INFO

Article history:
Received 22 December 2014
Received in revised form 1 April 2015
Available online 21 April 2015

Keywords:
Bistable microelectromechanical systems
Initially curved microbeam
Symmetric snap-through criterion
Electrostatic force
Boundary conditions
Arch shape
Residual axial force
Uniform temperature variation

ABSTRACT

In this paper, we extend the earlier studies to investigate the effects of various parameters which govern the symmetric snap-through buckling of an initially curved microbeam subject to an electrostatic force. The governing formulations are developed using Euler–Bernoulli beam theory. The mid-plane stretching experienced during the snap-through buckling is considered using von Karman nonlinear strain, and the nonzero strain component is determined and solved using Galerkin decomposition approach. The studied parameters include: beam fixation type (double-clamped and simply-supported), arch shape, residual axial force, and uniform temperature variation. The results of our work reveal the significant effects of the type of the beam fixation, the residual force, and the temperature variation on the criterion for the symmetric snap-through buckling of microbeams, while the effect of the arch shape is somewhat insignificant.

1. Introduction

In view of their potential usage as optical switches, micro-valves, and non-volatile memories (Charlot et al., 2008; Goll et al., 1996; Intaraprasonk and Fan, 2011; Ko et al., 2002; Roodenburg et al., 2009), the bistable Micro-Electro-Mechanical Systems (MEMS) based on initially curved microbeams have drawn considerable attention from the research community. The initially curved beam (i.e., arch) under transverse force may exhibit bistability, and the transition between two stable states is referred to as snap-through buckling (Medina et al., 2012).

A number of parameters govern the snap-through buckling, including the initial arch rise, the beam thickness, and the clamping angle. In 1990, Pippard conducted experiments to develop a phase diagram of instability in terms of the arch span and the initial angle at the clamped ends of the beam. This work was followed by Patricio et al. (1998) in which they developed a theoretical model to obtain a similar phase diagram. Based on the finite element simulations, Stanciulescu et al. (2012) showed that decreasing the beam temperature below a critical value eliminates the snap-through buckling. Following that, Virgin et al. (2014) conducted experiments of snap-through buckling at different temperatures, and Moghaddasie and Stanciulescu (2013) determined the boundaries separating different stability behaviors in terms of temperature variation and initial arch rise from numerical simulations.

The research work cited above studies the snap-through buckling of macrobeams under mechanical and/or thermal loadings. There are also studies on the microbeams actuated by the electrostatic or electromagnetic force. As a result of the earlier efforts, Krylov et al. (2008) revealed that the snap-through buckling of a microbeam under the electrostatic force occurs at large initial arch rise. Pane and Asano (2008) conducted energy analysis and further found that the existence of bistable states depends on the ratio between the initial arch rise and the beam thickness. Park and Hah (2008) conducted theoretical investigations and showed that the existence of bistable states also depends on the residual axial stress in the microbeam. Das and Batra (2009a) studied the pull-in instability and the snap-through buckling of a micro-arch under electric loadings, and derived a phase diagram of instability in terms of critical load and arch rise. They also developed a finite element model to study the transient snap-through behavior, and found that when the electric voltage is applied at a high rate, the snap-through buckling is suppressed (Das and Batra, 2009b). Ouakad and Younis (2010) simulated the nonlinear vibration of the micro-arch, and found that under certain DC and AC voltages, the arch may exhibit the dynamic snap-through buckling in the frequency range near the resonance frequency. Medina et al. (2012, 2014a, 2014b) examined, both theoretically and

* Corresponding author. Tel.: +1 416 978 5741.
E-mail address: meguid@mie.utoronto.ca (S.A. Meguid).

http://dx.doi.org/10.1016/j.ijsolstr.2015.04.011
0020-7683/

experimentally, the symmetric buckling and anti-symmetric bifurcation of an electrostatically actuated and initially curved mircobeams with/without residual stress. They derived the criteria of symmetric and non-symmetric snap-through instability for quasi-static loading conditions.

In this paper, we extend the earlier studies by developing the appropriate governing equations that lead us to systematically examine the influence of the parameters that govern the snap-through buckling of the initially curved microbeam actuated by electrostatic force. Such parameters include: type of beam fixation, arch shape, residual axial stress, and uniform temperature variation. Specifically, we study the influence of these parameters on the snap-through behavior (e.g., critical voltage at the snap-through buckling) and the derived snap-through criterion.

2. Governing equations

The situation envisaged is that of an initially curved microbeam of span L, width b and thickness h actuated by an electrostatic force (see Fig. 1). Suppose the respective displacements u_x, u_y and u_z along x, y and z coordinates only depend on x and z, and u_y is 0. It is further assumed that the microbeam is thin ($h << L$) to facilitate the application of the Euler–Bernoulli beam theory, viz.:

$$u_x(x,z) = u(x) - zw'(x) \tag{1a}$$

$$u_z(x,z) = w(x) \tag{1b}$$

where $u(x)$ and $w(x)$ are respectively the axial (along x-coordinate) and transverse (along z) displacements of a point on the mid-plane of the beam. During the snap-through buckling, the mid-plane stretching can be important. To consider this effect, the von Karman nonlinear strain is used, and the nonzero strain component is calculated as (Reddy, 2011):

$$\varepsilon_{xx}^{*} = u' - zw'' + \frac{1}{2}(w')^2 \tag{2}$$

Further consider the initial strain due to the initial deflection $w_0(x)$, we calculate the axial strain change ε_{xx} as:

$$\varepsilon_{xx} = u' - z(w'' - w_0'') + \frac{1}{2}\left((w')^2 - (w_0')^2\right) \tag{3}$$

Then the variation δU_{elas} of the elastic strain energy can be calculated as:

$$\delta U_{elas} = \int_0^L \int_S (\underline{\underline{\sigma}} : \delta\underline{\underline{\varepsilon}}) ds dx = \int_0^L \int_S (\sigma_{xx}\delta\varepsilon_{xx}) ds\, dx \tag{4}$$

where $\int_S ds$ is the integral over the cross section (y–z plane in Fig. 1). Replace ε_{xx} with Eq. (3), integrate the result by parts with respect to x, and we obtain:

$$\begin{aligned}\delta U_{elas} = &-\int_0^L N'\delta u dx - \int_0^L \left(M'' + (Nw')'\right)\delta w dx - N(0)\delta u(0) \\ &+ N(L)\delta u(L) - (M'(0) + N(0)w'(0))\delta w(0) + (M'(L) \\ &+ N(L)w'(L))\delta w(L) + M(0)\delta w'(0) - M(L)\delta w'(L)\end{aligned} \tag{5}$$

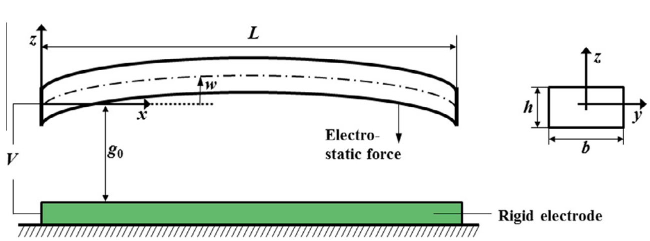

Fig. 1. Initially curved microbeam under electrostatic force (direction indicated by arrow).

where the normal force N and the bending moment M are defined as:

$$N = \int_S \sigma_{xx}\, ds \tag{6a}$$

$$M = \int_S z\sigma_{xx}\, ds \tag{6b}$$

The variation δW_{ext} of the work done by the distributed electrostatic force f_{elec} is:

$$\delta W_{ext} = \int_0^L (f_{elec}\delta w) dx \tag{7}$$

For a small gap (much smaller than the beam length) between the beam and the electrode, the beam with the electrode can be regarded as a parallel-plate capacitor. Further suppose that the beam is infinitely wide, then the fringing field effect can be neglected, and f_{elec} per unit length can be calculated as (Batra et al., 2007):

$$f_{elec} = -\frac{1}{2}\frac{\varepsilon_0 b V^2}{(g_0 + w)^2} \tag{8}$$

where ε_0 (=8.8542 × 10^{-12} F · m^{-1}) is the vacuum permittivity; g_0 is the gap between the beam ends and the electrode (see Fig. 1). Introduce Eqs. (5) and (7) into the theorem of minimum potential energy: $\delta U_{elas} - \delta W_{ext} = 0$, and we have:

$$\begin{aligned}&\int_0^L N'\delta u dx + \int_0^L \left(M'' + (Nw')' + f_{elec}\right)\delta w dx + N(0)\delta u(0) \\ &- N(L)\delta u(L) + (M'(0) + N(0)w'(0))\delta w(0) - (M'(L) \\ &+ N(L)w'(L))\delta w(L) - M(0)\delta w'(0) + M(L)\delta w'(L) = 0\end{aligned} \tag{9}$$

From Eq. (9), the following governing equations can be obtained:

$$\delta u : N' = 0 \tag{10a}$$

$$\delta w : M'' + (Nw')' + f_{elec} = 0 \tag{10b}$$

Suppose the beam material is elastically isotropic with Young's modulus E and Poisson's ratio ν. For an infinitely wide beam (beam width >> thickness), the effective elasticity modulus for the plane stress case (x–y plane in Fig. 1) can be used. Then the 1D constitutive relation becomes:

$$\sigma_{xx} = \frac{E}{1-\nu^2}\varepsilon_{xx} \tag{11}$$

Introduce Eqs. (3) and (11) into Eq. (6), and we obtain:

$$N(x) = \frac{ES}{1-\nu^2}\left(u' + \frac{1}{2}(w')^2 - \frac{1}{2}(w_0')^2\right) \tag{12a}$$

$$M(x) = -\frac{EI}{1-\nu^2}(w'' - w_0'') \tag{12b}$$

where S (=bh) is the cross-sectional area; I (=$bh^3/12$) is the second moment of area. Introduce Eqs. (8), (10a), and (12) into Eq. (10b), replace the normal force with an average value, and consider that the axial displacement is blocked at the two ends of the beam, we can obtain the following governing equation (Medina et al., 2012):

$$\begin{aligned}&\frac{EI}{1-\nu^2}(w'''' - w_0'''') - \frac{ES}{2(1-\nu^2)L}\left(\int_0^L ((w')^2 - (w_0')^2) dx\right) w'' \\ &+ \frac{1}{2}\frac{\varepsilon_0 b V^2}{(g_0 + w)^2} = 0\end{aligned} \tag{13}$$

Rewrite Eq. (13) in the non-dimensional form as (Medina et al., 2012):