International Journal of Solids and Structures 62 (2015) 39–51

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Imperfection-insensitive axially loaded thin cylindrical shells

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ARTICLE INFO

Article history:
Received 19 November 2013
Received in revised form 2 December 2014
Available online 7 February 2015

Keywords:
Shell buckling
Imperfection-sensitivity
Corrugated shells
Structural optimization
Mass efficiency
Aster shell

ABSTRACT

The high efficiency of circular monocoque cylindrical shells in carrying axial loads is impaired by their extreme sensitivity to imperfections and there is an extensive body of literature that addresses this behavior. Instead of following this classical path, focused on circular cross-sections, this paper presents a novel approach that adopts optimal symmetry-breaking wavy cross-sections (wavy shells). The avoidance of imperfection sensitivity is achieved by searching with an evolutionary algorithm for smooth cross-sectional shapes that maximize the minimum among the buckling loads of geometrically perfect and imperfect wavy shells. It is found that shells designed through this approach can achieve higher critical stresses and knockdown factors than any previously known monocoque cylindrical shells. It is also found that these shells have superior mass efficiency to almost all previously reported stiffened shells.

1. Introduction

Large discrepancies between analytically predicted and experimentally measured buckling loads for monocoque cylindrical shells were first observed in the 1930's and it was subsequently established that thin cylindrical shells under axial compression may buckle at loads as low as 20% of the classical value (Brush and Almroth, 1975). Hence, in practice empirically defined knockdown factors are used to decrease the theoretically estimated buckling loads of such shells and this is the currently accepted design approach. Therefore, monocoque cylindrical shells are designed for much larger theoretical buckling loads to ensure that, when the knockdown factor is applied, they still meet their design requirements (Jones, 2006).

The potential structural efficiency of monocoque cylindrical shells in carrying axial loads has been lost due to their extreme sensitivity to geometric imperfections, boundary conditions, loading, etc. and hence for all applications requiring the highest structural efficiency they have been replaced by an alternative structural architecture, the closely stiffened shell, a cylindrical shell reinforced by stringers/corrugations and rings. This alternative architecture is currently established as the premiere efficient aerospace structure (Singer et al., 2002) and is widely used for lightness and extreme efficiency.

We propose an alternative approach that builds on previous work by Ramm and co-workers (Reitinger and Ramm, 1995; Reitinger et al., 1994; Ramm and Wall, 2004), and consists in designing linear-elastic monocoque cylindrical shells with a special cross-sectional shape that maximizes the critical buckling load and at the same time reduces imperfection-sensitivity. These novel shells have asymmetric cross-section and their behavior is fundamentally different from shells designed with the knockdown-factor method.

A key feature of the proposed approach is that the critical buckling loads of both perfect and imperfect candidate designs are introduced in a structural optimization process (Reitinger et al., 1994; Reitinger and Ramm, 1995), and hence its outcome is a design that has a high buckling load and at the same time is also imperfection-insensitive. Standard optimization techniques focus only on maximizing the critical buckling load and tend to converge towards designs that are highly imperfection-sensitive, see for example Thompson, 1972. This serious drawback is avoided in the present approach. The optimization technique by Reitinger et al. (1994) and Reitinger and Ramm (1995) is applicable to any type of structure and hence can also be used to design imperfection-insensitive cylindrical shells with maximal critical buckling loads.

The paper is organized as follows. Section 2 presents the essential background to the present study. It includes: a brief literature review of the influence of imperfections on cylindrical shells; a review of current design approaches to avoid buckling; the selection of appropriate imperfections in buckling analysis; previous work on the design of imperfection-insensitive shells; and structural efficiency metrics for shell buckling. With this background, Section 3 presents a new methodology for the design of imperfection-insensitive cylindrical shells. The implementation in

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http://dx.doi.org/10.1016/j.ijsolstr.2014.12.030
0020-7683/

Section 4 produces four designs of carbon-fiber composite cylindrical shells. Section 5 analyzes these results in more detail and Section 6 considers two alternative design approaches. The mass efficiency of the new designs is then compared to existing stiffened shells in Section 7. Section 8 concludes the paper.

2. Background

There is a huge body of literature on the buckling of linear-elastic thin shells and the interested reader is referred to the extensive reviews compiled by many authors (Brush and Almroth, 1975; Elishakoff, 2012; Hutchinson and Koiter, 1970; Jones, 2006). This review is focused on the essential background to the present study.

2.1. Effects of imperfections on cylindrical shells

The first major contribution to the present understanding of the effects of initial imperfections on the buckling of circular cylindrical shells was made by Von Kármán and Tsien (1941) who analyzed the postbuckling equilibrium of axially compressed cylindrical shells. Donnell and Wan (1950) analyzed initially imperfect cylindrical shells and obtained equilibrium paths as sketched by the dash line in Fig. 1, where P and P_{cl} are the compressive load and the classical bifurcation buckling load, respectively. Fig. 1 shows a sharply dropping second equilibrium path and thus indicates that an initially imperfect shell buckles at the limit point B instead of reaching the bifurcation point A. Koiter (1963) analyzed the influence of axisymmetric imperfections coinciding with the axisymmetric buckling mode of a perfect cylindrical shell. His results, summarized in Fig. 2, show that imperfections with even a small amplitude can dramatically reduce the buckling load.

A more general analysis of the influence of initial imperfections (Koiter, 1945) was based on an analysis of the potential energy of the loaded structure in a general buckled equilibrium configuration. This analysis is applicable to asymmetric imperfections and shells of arbitrary shape (Brush and Almroth, 1975), and provides an approximate solution to the secondary equilibrium path for a perfect structure, with a single buckling mode associated with the first bifurcation point:

$$\lambda_0 \equiv \frac{P}{P_{cl}} = 1 + a_1\delta + a_2\delta^2 + \cdots \tag{1}$$

where $a_1, a_2, \ldots$ are constants and δ is a measure of the lateral displacement amplitude. This solution is shown by means of solid lines in Fig. 3. In case I $a_1 \neq 0$ and for small values of δ the secondary equilibrium path is approximated by a straight line. For the other two cases $a_1 = 0$, resulting in quadratic secondary equilibrium paths: $a_2 < 0$ for case II and $a_2 > 0$ for case III.

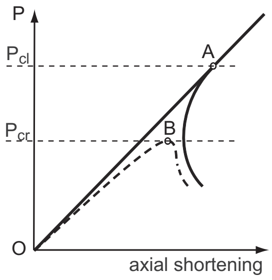

Fig. 1. Sketch of equilibrium paths for axially compressed, geometrically perfect cylindrical shells (solid line, from Von Kármán and Tsien (1941)) and imperfect cylindrical shells (dash line, from Donnell and Wan (1950)).

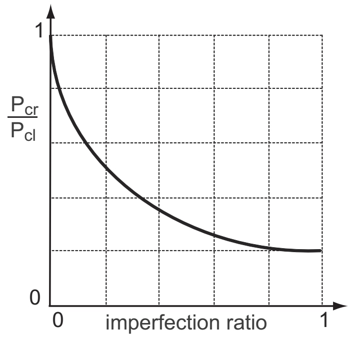

Fig. 2. Sketch of influence of imperfection amplitude (ratio of imperfection amplitude to shell thickness) on buckling load P_{cr} of imperfect shells, based on Koiter (1963).

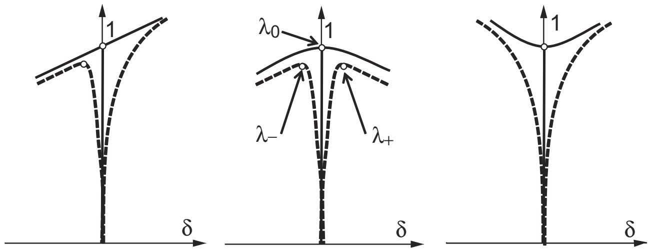

Fig. 3. Three types of post-buckling equilibrium paths for perfect and imperfect structures, from Brush and Almroth (1975) and Koiter (1945).

The corresponding equilibrium paths for imperfect structures are shown by dash lines in the figure. $\lambda_\pm$ are ratios between the buckling loads of imperfect structures with positive/negative imperfections and the perfect structure. Cases I and II represent structures that are *sensitive to imperfections*, because the buckling loads of the imperfect structures (λ_- for case I and $\lambda_\pm$ for case II) are lower than 1. In case I different signs of imperfections lead to different types of imperfection-sensitivity.

2.2. Design of cylindrical shells against buckling

The current approach for the design of axially compressed monocoque cylindrical shells against buckling accounts for buckling load reductions due to imperfections through the *knock-down-factor method*. The actual buckling load of a cylindrical shell is estimated from:

$$P_{cr} = \gamma P_{cl} \tag{2}$$

where γ is the knockdown factor and P_{cl} is given by Brush and Almroth (1975):

$$P_{cl} = \frac{2\pi E t^2}{\sqrt{3(1-\nu^2)}} \tag{3}$$

where E, ν and t are the Young's modulus, Poisson's ratio and shell thickness, respectively.

A widely used expression for γ is the empirical curve provided in NASA SP-8007 (Peterson et al., 1965) and shown in Fig. 4. Given the radius to thickness ratio R/t, this curve provides a lower bound to a large dataset of experimentally derived knockdown factors and hence can be used to predict the buckling load using Eq. (2).

Designs obtained from the knockdown factor method are required to achieve a theoretical buckling load P_{cl} high enough that the reduced buckling load P_{cr} obtained from Eq. (2) satisfies the design requirements. Fundamentally, the knockdown-factor design method accepts highly imperfection-sensitive shell designs, but