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Hygro-elasto-plastic model for planar orthotropic material

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ABSTRACT

An in-plane elasto-plastic material model and a hygroexpansivity-shrinkage model for paper and board are introduced in this paper. The input parameters for both models are fiber orientation anisotropy and dry solids content. These two models, based on experimental results, could be used in an analytical approach to estimate, for example, plastic strain and shrinkage in simple one-dimensional cases, but for studies of the combined and more complicated effects of hygro-elasto-plastic behavior, a numerical finite element model was constructed. The finite element approach also offered possibilities for studying different structural variations of an orthotropic sheet as well as buckling behavior and internal stress situations under local strain differences. A few examples are presented of the effect of the anisotropy and moisture streaks under stretching and drying conditions on strain differences and buckling. The internal stresses were studied through a case in which the drying of different layers occurred at different stages. Both the anisotropy and moisture streaks were capable of rendering the buckling of the sample visible. The permanency of these defects highly depends on several process stages and tension conditions of the sheet, as demonstrated in this paper. The application possibilities of the hygro-elasto-plastic model are diverse, including investigation into several phenomena and defects appearing in drying, converting and printing process conditions.

1. Introduction

Paper and board have elasto-visco-plastic properties, exhibiting such rheological behaviors as delayed strain recovery, stress relaxation and creep (Skowronski and Robertson, 1986; Rance, 1956; Steenberg, 1947; Gates and Kenworthy, 1963; Lyne and Gallay, 1954). The hygroscopic nature of the pulp fibers causes dimensional changes in the paper or the board when subject to influences of humidity changes or treatments which involve water intake or drying (Rance, 1954; Page and Tydeman, 1962; Uesaka, 1994; Nanko and Wu, 1995). Shrinkage during drying (Wahlström and Lif, 2003; Hoole et al., 1999; Nanri and Uesaka, 1993; Kiyoaki, 1987) and hygro- and hydroexpansivity (Uesaka, 1991; Salmen et al., 1987; Larsson and Wagberg, 2008; Lif et al., 1995; Mendes et al., 2011) are widely studied components of sorption based dimensional instabilities. Natural fibers and their treatments, bonds between fibers and their orientation in the fiber network, additives and manufacturing conditions all affect dimensional instability and mechanical properties of paper or board (Silvy, 1971; Wahlström and Fellers, 2000; Alava and Niskanen, 2006; Kouko et al., 2007; Nordman, 1958; Fahey and Chilson, 1963; Salmen et al., 1987; Uesaka et al., 1992; Uesaka and Qi, 1994; Mäkelä, 2009; Lyne et al., 1996; de Ruvo et al., 1976; Manninen et al., 2011; Setterholm and Kuenzi, 1970; Glynn et al., 1961; Leppänen et al., 2008).

Several models to predict in-plane mechanical and rheological properties, shrinkage and hygroexpansivity have been introduced in the literature. Johnson and Urbanik (1984) and Johnson and Urbanik (1987) have provided a nonlinear elastic model to study material behavior in stretching, bending and buckling of axially loaded paperboard plates. Nonlinear elastic biaxial failure criteria have been studied by Suhling et al. (1985) and Fellers et al. (1983). In-plane orthotropic elasto-plastic approaches to estimate the tensile response and deformation of paper have been presented by Castro and Ostoja-Starzewski (2003), Mäkelä and Östlund (2003) and Xia et al. (2002). Viscoelastic models have been used extensively in studying creep or relaxation behavior (Brezinski, 1956; Lif et al., 1999; Lu and Carlsson, 2001; Pecht et al., 1984; Pecht and Johnson, 1985; Rand, 1995; Uesaka et al., 1980).

A formula for the hygroexpansion of paper relating to the hygroexpansion of a single fiber and the efficiency of the stress transfer between fibers has been derived by Uesaka (1994). The traditional theory of linear thermoelasticity was applied to

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estimate hygroexpansion strains in the study of Lavrykov et al. (2004). Hygro-viscoelastic models have been applied to estimate history dependent dimensional stability and hygroexpansivity by Uesaka et al. (1989), Uesaka (1991), Lif et al. (2005) and Lif (2006). Mechano-sorptive creep has been studied, for example, by Urbanik (1995), Strömbro and Gudmundson (2008), Alfthan (2004) and Haslach (1994). Shrinkage profiles have been modeled by Wahlström et al. (1999) and Constantino et al. (2005). The measured moisture dependency of material constants has been employed within the nonlinear elastic model for an investigation of the effect of moisture on mechanical behavior by Yeh et al. (1991). Hygroscopic out-of-plane deformations, such as curling and buckling, have been studied, using elastic constitutive models, by Bloom and Coffin (2000), Leppänen et al. (2005) and Kulachenko et al. (2005); while an elasto-plastic model has been introduced in Lipponen et al. (2008, 2009). The hygroexpansion coefficients are independent of moisture content in these models.

In this paper, an in-plane elasto-plastic material model and a hygroexpansivity-shrinkage model that are functions of the dry solids content and fiber orientation anisotropy index are introduced. The elasto-plastic model is based on fittings of the uni-axial stress–strain curves presented by Lipponen et al. (2008) and Erkkilä et al. (2013). The dependency of the dry paper hygroexpansion coefficient on the anisotropy index was determined using the measurement results presented in Erkkilä et al. (submitted for publication). The drying shrinkage strain as a function of the dry solids content was constructed with an exponential formula based on the measurements provided by Ivarsson (1954), Kijima and Yamakawa (1978) and Tydeman et al. (1966) and summarized by Wahlström et al. (1999) and Wahlström (2004). The relation between the hygroexpansivity and the solids content was derived from the drying shrinkage strain function. Usually, the hygroexpansivity has been considered to be constant (i.e. independent of the moisture content level), which estimates the change in dimensions of a dry paper subject to the relative humidity change. However, in this paper, the dry solids content dependent hygroexpansivity, over the entire range from wet to dry, has been introduced. These two models, the elasto-plastic material model and the hygroexpansivity-shrinkage model, were exploited when numerical solutions were obtained with the finite element method. The use of the anisotropy index instead of that of traditional fiber orientation anisotropy simplified the handling of different in-plane directions in the case of the anisotropic sheet and, for instance, the determination of Hill's yield surface for the finite element approach is straightforward. Analytical one-dimensional solutions were used to estimate plastic and hygroscopic strains in simple cases and the results were compared with numerical simulations.

2. Models

In this section, the one-dimensional elasto-plastic material model (Section 2.1) and the hygroexpansivity-shrinkage model (Section 2.2) are presented. The continuum mechanical model is constructed in Section 2.3 and the numerical solution approach is described in Section 2.4.

2.1. Elasto-plasticity

The stress–strain measurements, used here as the basic data for constructing the material model, were presented in Lipponen et al. (2008) and Erkkilä et al. (2013). The stress–strain curve fittings for the determination of the elastic modulus, yield strain, yield stress and function for the strain hardening behavior were discussed in detail in Erkkilä et al. (2013). The following equation was considered suitable for describing all the uniaxial stress–strain relationships studied:

$$\sigma = \begin{cases} E\varepsilon & \text{if } \varepsilon \leqslant \varepsilon_y \\ E\varepsilon_y - \frac{H}{2E} + \sqrt{H\left(\frac{H}{4E^2} + \varepsilon - \varepsilon_y\right)} & \text{if } \varepsilon > \varepsilon_y \end{cases} \quad (1)$$

where σ is the stress and ε is the strain; the elastic modulus E, the yield strain ε_y and the hardening constant H are the fitting parameters. The fitting parameters were determined for different dry solids contents (R_{sc}) and anisotropy index (ϕ) levels from the materials measured in Erkkilä et al. (2013). To construct the material model, the following equation was used to fit the parameters $\sigma_y,\ \varepsilon_y$ and H as functions of R_{sc} and ϕ:

$$P = (A_1 + A_2\phi + A_3R_{sc})^{1/n} \quad P = \{\sigma_y, \varepsilon_y, H\} \quad (2)$$

where $A_1,\ A_2,\ A_3$ and n are the fitting constants listed in Table 1. In Erkkilä et al. (2013) the anisotropy index was defined as:

$$\phi = \sqrt{\frac{1-\xi^2}{\xi + \tan^2\gamma/\xi} + \xi} \quad (3)$$

where ξ is the fiber orientation anisotropy and γ is the angle from the minor axis of the fiber orientation distribution. The anisotropy index in the main direction and cross direction (direction perpendicular to the main direction) obtain values of $\sqrt{\xi}$ and $1/\sqrt{\xi}$, respectively. Table 1 also includes the coefficient of determination r^2 values between the measured parameters from Erkkilä et al. (2013) and their estimates according to Eq. (2). The elastic modulus is determined by $E = \sigma_y/\epsilon_y$, with $r^2 = 0.985$. The material model parameters as a function of ϕ and R_{sc} are presented in Fig. 1. Since the functions fitted according Eq. (2) behave monotonically a reasonable amount of extrapolation is permitted. This may be needed, for example, if the local variation of the fiber orientation is considered. The model has a lower limit for solids content; i.e., it is valid if $R_{sc} > 0.3$; the parenthetical expression of Eq. (2) reaches negative values for yield stress σ_y with low solids content if the anisotropy index is also simultaneously low. The material model can be used directly to calculate the material parameters of the orthotropic sample in any in-plane direction or at any solids content level. Examples of the determination of plastic strain as a consequence of stretching either 0.3% or 1% and releasing afterwards are presented in Fig. 2.

2.2. Hygroexpansivity and shrinkage

The hygroexpansivity-shrinkage model is based on the measured relationships between the dry paper hygroexpansivity β_d and anisotropy index ϕ

$$\beta_d = k\phi^{\nu} \quad (4)$$

and between the drying strain ε_{ds} and the dry paper hygroexpansivity

$$\varepsilon_{ds} = -\frac{1}{a}\beta_d + \frac{b}{a} \quad (5)$$

where $k,\ \nu,\ a$ and b are fitted constants for freely-dried (fd) and restraint-dried (rd) samples made of either softwood pulp (SW), thermomechanical pulp (TMP) or a mixture of those (MIX); see

Table 1
The fitting parameters of Eq. (2) and the coefficients of determination r^2 for the yield stress σ_y, the yield strain ε_y and the hardening constant H.

	A_1	A_2	A_3	n	r^2
σ_y	−5.9030 (Pa^n)	3.1959 (Pa^n)	18.3077 (Pa^n)	0.1760 (–)	0.965
ε_y	380.4181 (–)	14.3408 (-)	−269.8327 (–)	−0.7720 (–)	0.816
H	−0.6021 (Pa^{2n})	4.0423 (Pa^{2n})	11.3795 (Pa^{2n})	0.0715 (–)	0.890