International Journal of Solids and Structures 62 (2015) 124–133

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Mechanics of wrinkling of a thin film bonded to a compliant substrate under the influence of spatial thermal modulation

Shrinidhi S. Pandurangi [1], Salil S. Kulkarni *

Department of Mechanical Engineering, IIT Bombay, Powai, Mumbai 400076, India

ARTICLE INFO

Article history:
Received 4 November 2014
Received in revised form 1 February 2015
Available online 20 February 2015

Keywords:
Thin film/substrate
Stability
Spatial thermal modulation

ABSTRACT

A thin film bonded to an elastic substrate and subjected to compressive in-plane stresses may develop wrinkles. In this paper a theoretical model of formation of one-dimensional sinusoidal wrinkles in such a film/substrate system subjected to a commensurate spatially varying temperature field is first presented and then analyzed. It is shown that an initially flat thin film on an elastic substrate can develop controlled wrinkles when subjected to a carefully chosen spatial thermal modulation.

1. Introduction

Wrinkling of stiff thin films on soft substrates is a common phenomenon in nature. In engineering systems a stiff thin film bonded to a soft compliant substrate may develop spatially uniform in-plane residual stresses during the process of cooling down from the deposition temperature due to the mismatch in the coefficient of thermal expansion between the film and the substrate. The flat film becomes unstable and is observed to form wrinkles if the residual stresses in the film reach a critical compressive stress required for the onset of buckling. Several theoretical models have been developed in the existing literature to explain this wrinkling process (Audoly and Boudaoud, 2008; Chen and Hutchinson, 2004; Groenewold, 2001; Huang, 2005; Huang et al., 2005; Mahadevan and Rica, 2005). Without any control over the wrinkle formation, the wrinkles in engineering structures can be a nuisance. They can degrade the reliability of integrated structures (Iacopi et al., 2003). Wrinkle formation can act as a failure mechanism causing roughening of thermal barrier coatings of turbine blades under cyclic temperatures (Clarke and Levi, 2003).

With control, the wrinkle forming mechanisms can be used to fabricate patterned surfaces especially in nano and micro-mechanical and electronic systems (Schweikart and Fery, 2009). Wrinkles can be used as stretchable interconnects in large-area flexible electronic circuits (Lacour et al., 2004). By tailoring the wrinkle orientation and geometry, they can be effectively harnessed to design better photonic structures which would help in increasing the efficiency of solar panels (Kim et al., 2012). The phenomenon of wrinkling can be used as an application in metrology for determining the elastic properties of materials (Schweikart and Fery, 2009; Genzer and Groenewold, 2006).

One possible mechanism of controlling such pattern forming systems is spatial periodic forcing. The control in such periodically forced systems is achieved due to the phenomenon of wavenumber locking. The effect of spatial periodic forcing on pattern formation has been studied in many different systems, e.g. hydro-dynamical systems (Kelly and Pal, 1978; Lowe et al., 1983), nonlinear optics (Neubecker and Jakoby, 2003), etc. With regards to the thin film/substrate under consideration, recent experiments (Kim et al., 2010) show that wrinkles can be developed on a selected area of the film by introducing local thermal modulations. Further, it was shown that a spatially periodic thermal modulation imposed over the system can influence the flat film to form periodic wrinkles. The present theoretical work is motivated by the experiments carried out in this regard. Through this work an attempt has been made to investigate the existence of conditions that could make a uniformly stressed flat film lose stability and form translationally invariant one dimensional wrinkles in the presence of spatially periodic temperature perturbations. In the absence of any modulation, the periodicity of the wrinkles is invariant under spatial translation. In order to maintain this spatial invariance, only commensurate modulations (Coullet, 1986) are considered in the paper. In our earlier work (Pandurangi and Kulkarni, 2014), preliminary results for one special case of commensurate modulation have been discussed.

* Corresponding author. Tel.: +91 2225767513.
E-mail addresses: pshrinidhi.269@gmail.com (S.S. Pandurangi), salil.kulkarni@iitb.ac.in (S.S. Kulkarni).

[1] Current address: GE India Technology Centre Pvt. Ltd., Bangalore 560066, India.

http://dx.doi.org/10.1016/j.ijsolstr.2015.02.019
0020-7683/

The outline of the paper is as follows. The stability analysis of a flat film bonded to an infinitely thick substrate in the absence of thermal modulations is summarized in Section 2. In Section 3 the effect of thermal modulations is incorporated in modeling the wrinkling phenomenon. Finally, the summary and conclusions are presented in Section 4.

2. Wrinkling in the absence of thermal modulation

The primary reference for the film/substrate model is as considered in Audoly and Boudaoud (2008). Fig. 1 shows the model under consideration consisting of an elastic thin film of thickness h bonded on a thick elastic substrate. The film and the substrate extend infinitely in the directions x_1 and x_2 allowing plane strain conditions to be considered far from the domain boundaries. The film is stiff in comparison to the substrate. The substrate complies with the film during wrinkling so that the film does not de-bond from the substrate surface during the process of deformation. In the absence of an external temperature field, the film wrinkles under the action of uniform residual compressive stresses only.

The film, in its unbuckled configuration, is subjected to a state of spatially uniform in-plane residual compressive strains $(-\epsilon^r_{11})$ and $(-\epsilon^r_{22})$, $(\epsilon^r_{11} > 0,\ \epsilon^r_{22} > 0)$ which denote the principal strains in the film aligned along x_1 and x_2 axes. They are attributed to differential expansion undergone by the film and the substrate cooling down from the deposition temperature to a temperature T_i. The film is assumed to be homogeneous and isotropic with Young's modulus and Poisson's ratio, E_f and ν_f, respectively. The substrate is assumed to be homogeneous and isotropic with Young's modulus E_s and Poisson's ratio ν_s and undergoes small deformation. For the flat configuration of the film, the residual strains in the substrate are assumed to be zero. The substrate is assumed to be semi-infinite in the negative x_3 direction, with the substrate being bonded to the film at $x_3 = 0$.

The flat film is considered as the reference configuration. In its deformed configuration, it is assumed that the film undergoes moderate rotation such that the wavelength of the wrinkled film is much larger than its thickness. The kinematics of the film deformation are modeled using Von-Karman plate equations. Zero in-plane displacement boundary conditions are assumed at the film/substrate interface. Stability analysis based on finding the equilibrium configuration by minimizing the total strain energy of the film/substrate system indicates that the critical compressive stress $|\sigma^r_{11}|^*$ required for the flat film to lose stability and form wrinkles is given by Audoly and Boudaoud (2008)

$$\frac{|\sigma^r_{11}|^*}{\bar{E}_f} = \left[\frac{3\theta E_s}{4\bar{E}_f}\right]^{2/3} \tag{1}$$

where, $\bar{E}_f = E_f/(1-\nu_f^2)$ is the modified Young's Modulus of the film and θ is a dimensionless function of the substrate's Poisson ratio given by

$$\theta = \frac{2(1-\nu_s)}{(1+\nu_s)(3-4\nu_s)} \tag{2}$$

The equilibrium wavelength λ_{eq} and equilibrium amplitude A_{eq} of the wrinkles are given by Audoly and Boudaoud (2008)

$$\frac{1}{k_{eq}h} = \frac{\lambda_{eq}}{2\pi h} = \left[\frac{\bar{E}_f}{6\theta E_s}\right]^{\frac{1}{3}} \tag{3}$$

and

$$\frac{A_{eq}}{h} = \sqrt{\frac{2}{3}}\sqrt{\left[\frac{|\sigma^r_{11}|}{|\sigma^r_{11}|^*} - 1\right]} \tag{4}$$

Here, $|\sigma^r_{11}| = \bar{E}_f(\epsilon^r_{11} + \nu_f \epsilon^r_{22})$ represents the residual compressive stress in the film.

Next, the effect of the spatial thermal modulation on the wrinkling phenomenon will be investigated.

3. Wrinkling under thermal modulation

The film/substrate system (see Section 2) with identical assumptions with regards to the geometry and the material properties is considered for the present analysis. The film and the substrate are assumed to be at an initial temperature T_i as before. Assuming that the state of uniform initial residual compressive stress in the film is below the critical buckling stress, a spatially periodic temperature perturbation is introduced in the system. Under the effect of the imposed temperature field, the film and the substrate are assumed to deform in x_1x_2 plane. The magnitude of the perturbation is assumed to be small such that the film and the substrate undergo compliant elastic deformations only. It is also assumed that the moduli of elasticity, the Poisson's ratios and thermal expansion coefficients do not vary in the imposed temperature range. Finally, an equilibrium solution of the form of $w(x_1) = A_{eq}\cos(k_{eq}x_1)$ is sought by formulating the problem as a strain energy minimization problem. Here $w(x_1)$ is the out-of-plane displacement and A_{eq} and k_{eq} are the equilibrium amplitude and the equilibrium wavenumber, respectively.

3.1. Thermo-elastic analysis of the film

As before, one considers a flat film in the x_1x_2 plane subjected to uniform in-plane compressive residual strains, $(-\epsilon^r_{11})$ and $(-\epsilon^r_{22})$ along x_1 and x_2 directions, respectively. A thermal modulation is imposed on the film surface in the x_1 direction. The effect of this modulation is to perturb the initial temperature of the flat film such that,

$$T_f = T_i + T_o\cos(k_2x_1), \quad T_o > 0 \tag{5}$$

Here k_2 is the wavenumber of thermal modulation. The temperature difference due to modulation is denoted by $\Delta T = T_o\cos(k_2x_1)$ and is assumed to be constant throughout the film

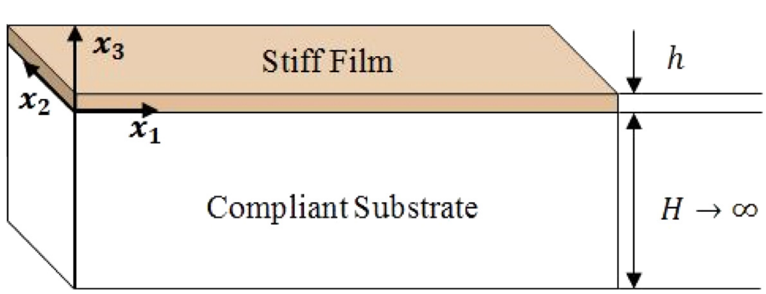

Fig. 1. Schematic of the film/substrate system.