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Nondimensional analysis of single lap joint subjected to out-of-plane loading

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ABSTRACT

Transverse deflection of a single lap joint subjected to both in-plane and out-of-plane loadings is analyzed, incorporating nondimensional analysis. Solution from such analysis can then be exploited in obtaining the energy release rate and global mode mixity of a cracked single lap joint. Being contingent on debonding propensity of the crack tips, the kinetic energy absorption capacity in the event of low-velocity impact is examined based on global energy-balance consideration. Results reveal that calculations of change in impact energy level in experiment, as well as energy release rate and mode mixity in numerical models are comparable with nondimensional model. Though a gain in energy release rate is expected, the presence of in-plane loading also appears to improve the impact energy absorption capacity at high kinetic energy level. For a reasonably design estimate of low-velocity impact behavior of a single lap joint, the nondimensional analysis presented is considerably less cumbersome to handle.

1. Introduction

Single lap joints have been widely studied as a joining configuration for adhesively bonded joints. Its simplicity in adherend preparation renders its prevalence in adhesively bonded design for structural applications. The use of polymer matrix composites in aerospace and automotive industries has driven the use of adhesively bonded technique over conventional mechanical joining methods such as bolted and riveted connection, in consideration of reduction in stress concentration and occurrences of manufacturing defects triggered by the fastener holes. The debonding problem of single lap joint has been traditionally approached by assessment of peel and shear stresses at the adhesive bondline. This so-called stress-based approach has its root in early classical analyzes carried out by Volkersen (1938) as well as Goland and Reissner (1944), and further modified and refined by several authors. A summary of these analytical models is presented by da Silva et al. (2009) recently. The same debonding problem at the bondline has also been approached by means of linear-elastic fracture mechanics (Anderson et al., 1988; Fernlund et al., 1994; Shahin and Taheri, 2008; Luo and Tong, 2009). Predicting failure using this energy-based approach has its assumption in that macroscopic crack exists such that the single lap joint is inherently cracked and its fracture load is independent whether such macrocrack exist or not in reality (Fernlund et al., 1994).

Substantial studies in single lap joint have mainly focused on the in-plane loading at both ends of adherends, as such joint is primarily configured to take up tension loading. In practice, there are occasions in which the single lap joint is subjected to external transverse deflection (hence also an out-of-plane loading) typically at the overlap region. A direct result of this transverse deflection is the rising of the external bending moments that would increase the severity of peel stress concentration at the debonding edge. Limited studies on transverse deflection problem of single lap joint are available in literature, where high-velocity impact of composite joint is more emphasized among those studies. High-velocity impact onto composite joints requires much attention as the damage could additionally involve the delamination of composite panel, apart from debonding at the adhesive bondline. Kim and Kedward (2000) found that initial response of high-velocity impact of composite panel is independent of boundary condition, where panel deflection remain localized, due to predominance of stress wave propagation. However, an investigation by Park and Kim (2010) on the high-velocity impact of composite single lap joints by spherical ice presented few experimental evidences that

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support the favorable initiation of damage caused by stress concentration of debonding edge, instead of localized damage normally observed in impact of composite panel alone.

Investigations involving low-velocity impact were also conducted in regard to composite single lap joints. The finite element analysis conducted by Higuchi et al. (2002) reported that the maximum principal stress of a single lap joint subjected to impact bending increases with increases in adherends' Young's modulus and overlap length. This trend is observed to be contradictory compared to static analysis carried by Liu et al. (1998). Vaidya et al. (2006) performed a quasi-static stress analysis on the carbon/epoxy composite single lap joints under in-plane and transverse loadings. Their results suggested that cohesive failure of bondline is correlated to higher peel stress concentration sustained at joint edge, when the joint is transversely loaded. The effect of point impactor, which represents projectile configuration encountered at the contact point of high-velocity impact, on the low-velocity transverse response of glass/epoxy bonded lap joint, was studied by Kim et al. (2005). Their experimental testing showed that complete global debonding occurs when the impactor kinetic energy increases to certain threshold level, whereby the failure is correlated to higher peel stress evaluated at the joint's free edge.

To the best of authors' knowledge, only Pang et al. (1995) had attempted to obtain the analytical treatment of transversely impacted single lap joint from a quasi-static equilibrium standpoint. They studied the impact force using the spring-mass model of Shivakumar et al. (1985). The shape of the experimental impact force versus time response curves is very similar to those predicted by spring-mass model. Studies by Park and Kim (2010) similarly give this inverted "U" shape experimental force history. A good correlation between the spring-mass and energy-balance models in Shivakumar et al. (1985) seems to suggest an analysis using energy-balance model may be equally attainable. This paper explores to present the two-dimensional transverse deflection problem using fracture mechanics approach in order to understand the debonding behavior of a single lap joint subjected to in-plane as well as out-of-plane loading. And the low-velocity impact event is modeled by considering the global energy balance. By adopting the methodology introduced by Lai et al. (1996), the paper demonstrates how the quasi-static deflection problem can be nondimensionalized and subsequently presents a nondimensionalized solution, to further describe the debonding propensity at peel stress concentration edge. The paper continues to illustrate how the insight into the debonding might be extended to serve as a reference in order to understand the kinetic energy absorption capacity of single lap joint subjected to low-velocity impact. An experimental investigation into the three-point bending of glass/epoxy composite single lap joint is undertaken to validate the general applicability of the developed analytical solution.

2. Problem description

2.1. Transverse deflection

The analysis presented in this paper build on earlier nondimensional approach by Lai et al. (1996), which focused on the cracked lap shear joint. A macroscopic cracked single lap joint subjected to both in-plane and out-of-plane loads is attempted herein. Fig. 1(a) illustrates the configuration of such single lap joint and its related parameters to describe the transverse deflection problem. At the overlap section, two crack fronts, respectively denoted as left-hand crack tip and right-hand crack tip, exist such that they usually represent the possible critical initiation sides of debonding or interfacial failure. Very thin adhesive layer, if any, compared to adherend thickness is assumed so that the analysis is not significantly affected. The actual length of the bonded overlap section, i.e. the distance between the two crack fronts l_B, is generally referred as *overlap length*.

The sign conventions of the in-plane load P, out-of-plane load F_T, bending moment M, and shear force R are labelled in Fig. 1(b), and the global force equilibrium requirement is concurrently depicted. P, F_T, M, and R are forces defined as per unit length, which all are distributed in the direction normal to the surface of the paper plane. It is more prudent to divide the deflection problem into four segments having their own local coordinate systems indicated in Fig. 1(c) as (x_1, w_1), (x_{01}, w_{01}), (x_{02}, w_{02}) and (x_2, w_2). w_1, w_{01}, w_{02} and w_2 represent the transverse deflections of the joint's neutral plane along x_1, x_{01}, x_{02} and x_2 axes. w_d is the deflection somewhere along the overlap length as a result of out-of-plane load F_T. Differential equations (1)–(4) respectively govern the segments' transverse deflections:

$$\frac{d^2 w_1}{dx_1^2} = -\frac{M_1}{D_1} = -\frac{P}{D_1}\left(\alpha_n x_1 - w_1 + \frac{R_{Ae}}{P}x_1 + \frac{M_A}{P}\right) \tag{1}$$

$$\frac{d^2 w_{01}}{dx_{01}^2} = -\frac{M_{01}}{D_0} = -\frac{P}{D_0}\left(\alpha_n(x_{01}+l_A) - w_{01} - \left(\frac{h_1}{2}+h_2-\Delta h_1\right) + \frac{R_A}{P}(x_{01}+l_A) + \frac{M_A}{P}\right) \tag{2}$$

$$\frac{d^2 w_{02}}{dx_{02}^2} = -\frac{M_{02}}{D_0} = -\frac{P}{D_0}\left(\alpha_n(x_{02}+l_d) - w_{02} - \left(\frac{h_1}{2}+h_2-\Delta h_1\right) + \frac{R_A}{P}(x_{02}+l_d) - \frac{F_T}{P}x_{02} + \frac{M_A}{P}\right) \tag{3}$$

$$\frac{d^2 w_2}{dx_2^2} = -\frac{M_2}{D_2} = -\frac{P}{D_2}\left(\alpha_n(x_2+l_A+l_B) - w_2 - \frac{1}{2}(h_1+h_2) + \frac{R_A}{P}(x_2+l_A+l_B) - \frac{F_T}{P}(x_2+l_A+l_B-l_d) + \frac{M_A}{P}\right) \tag{4}$$

Note that this paper follows the same notation used by Lai et al. (1996) in these equations where appropriate. Among these symbols: D_1, D_0 and D_2 are respectively the flexural rigidities of the first adherend, bonded overlap section, and second adherend, α_n is the angle subtended between the x_1 (or x_2) axis and in-plane load axis: $\alpha_n = (h_1+h_2)/2L$, Δ is a nondimensional quantity expressed as

$$\Delta = \frac{1+2\Sigma\eta+\Sigma\eta^2}{2\eta(1+\Sigma\eta)}$$

whereby η is defined thickness ratio of the first adherend to second adherend: $\eta = h_1/h_2$, Σ is defined as modulus ratio of the first adherend to second adherend: $\Sigma = E_1^*/E_2^*$, where $E_1^* = E_1/(1-\nu_1^2)$ and $E_2^* = E_2/(1-\nu_2^2)$ for the plane strain problem, or $E_1^* = E_1$ and $E_2^* = E_2$ for the plane stress problem.

Closed form solutions of these differential equations are respectively given as Eqs. (5)–(8).

$$\frac{w_1}{L} = A_1\cosh(\lambda_1 x_1) + B_1\sinh(\lambda_1 x_1) + (\alpha_n + \hat{R}_A)\frac{x_1}{L} + \hat{M}_A \tag{5}$$

$$\frac{w_{01}}{L} = A_{01}\cosh(\lambda_0 x_{01}) + B_{01}\sinh(\lambda_0 x_{01}) + (\alpha_n + \hat{R}_A)\frac{(x_{01}+l_A)}{L} - \frac{1}{L}\left(\frac{h_1}{2}+h_2-\Delta h_1\right) + \hat{M}_A \tag{6}$$

$$\frac{w_{02}}{L} = A_{02}\cosh(\lambda_0 x_{02}) + B_{02}\sinh(\lambda_0 x_{02}) + (\alpha_n + \hat{R}_A - \hat{F}_T)\frac{x_{02}}{L} + (\alpha_n + \hat{R}_A)\frac{(x_{02}+l_d)}{L} - \frac{1}{L}\left(\frac{h_1}{2}+h_2-\Delta h_1\right) + \hat{M}_A \tag{7}$$