International Journal of Solids and Structures xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A micropolar peridynamic theory in linear elasticity

Shubhankar Roy Chowdhury, Md. Masiur Rahaman, Debasish Roy *, Narayan Sundaram

Department of Civil Engineering, Indian Institute of Science, Bangalore 560012, India

ARTICLE INFO

Article history:
Received 1 October 2014
Received in revised form 17 January 2015
Available online xxxx

Keywords:
Micropolar peridynamics
Length scales
Constitutive correspondence
Timoshenko-type beams
Plane stress problems

ABSTRACT

A state-based micropolar peridynamic theory for linear elastic solids is proposed. The main motivation is to introduce additional micro-rotational degrees of freedom to each material point and thus naturally bring in the physically relevant material length scale parameters into peridynamics. Non-ordinary type modeling via constitutive correspondence is adopted here to define the micropolar peridynamic material. Along with a general three dimensional model, homogenized one dimensional Timoshenko type beam models for both the proposed micropolar and the standard non-polar peridynamic variants are derived. The efficacy of the proposed models in analyzing continua with length scale effects is established via numerical simulations of a few beam and plane-stress problems.

1. Introduction

Classical continuum mechanics assumes a continuous distribution of matter throughout the body and establishes the equations of motion considering only the local action. Both long range effects of loads and those of inter-molecular interactions are ignored in this theory. Such approximations limit the applicability of the classical theory to macro-scale phenomena where the characteristic length scale of the loading is much larger than the intrinsic material length scales. However, when these length scales are comparable, the microstructural effects could become significant and predictions of the classical theory depart considerably from experimental results. Substantial discrepancies are observed, for instance, in problems involving high stress gradients at notch and crack tips, short wavelength dynamic excitations, the behavior of granular solids, porous materials, modern-day engineering nano-structures etc. (Eringen, 1976).

In order to circumvent such limitations of the classical continuum theory of elasticity, an early attempt was made by Voigt (1887), who postulated the existence of a couple-traction along with the usual force-traction responsible for the force transfer across boundaries/interfaces or from one part of the body to another. Later Cosserat and Cosserat (1909) developed a mathematical model based on couple stresses leading to a description of the stress fields via asymmetric tensors as opposed to the symmetric Cauchy stress fields in the classical theory. From a kinematical perspective, this theory enables non-local interactions via the incorporation of rotational degrees of freedom, along with the classically employed translational ones, for the material points and this allows an infinitesimal volume element about a material point to rotate independently of the translational motion. This idea, as formalized in the micropolar theory (Eringen, 1999), assumes the material micro-rotation to be independent of the continuum macro-rotation (e.g. the curl of the displacement field). Such a microstructure-motivated description of deformation provided for the inclusion of length scale parameters in the constitutive equations which were otherwise absent. The development of a structured generalized continuum theory only took place several decades later. Among numerous such contributions, we cite (Eringen and Suhubi, 1964; Nowacki, 1970; Kafadar and Eringen, 1971; Eringen and Kafadar, 1976; Eringen, 1999) and the references therein.

Gradient type non-local formulations (Mindlin, 1965; Mindlin and Eshel, 1968) are yet another approach to a generalized continuum theory where, in lieu of micro-rotations, several higher order gradients of the strain tensor are assumed to contribute to the internal work thereby bringing in the length scale effect. As a consequence, such formulations introduce different higher order generalized stresses conjugate to the gradients of strain.

Apart from the limitation related to scale independence, it is known that the mathematical setup in the classical continuum mechanics may not be quite appropriate in the context of several other problems of fundamental interest in solid mechanics, viz. those including the spontaneous generation of cracks (Silling, 2000). The inability to track and evolve a discontinuous field may be traced back to the kinematical requirement of a sufficiently smooth, diffeomorphism-type deformation field appearing in the

* Corresponding author.
E-mail address: royd@civil.iisc.ernet.in (D. Roy).

http://dx.doi.org/10.1016/j.ijsolstr.2015.01.018
0020-7683/

governing partial differential equations (PDEs). Therefore computational methods for solving such problems using the classical theory either require a redefinition of the object manifold so that discontinuities lie on the boundary or some special treatment to define the spatial derivatives of the field variables on a cracked surface (Bittencourt et al., 1996; Belytschko and Black, 1999; Areias and Belytschko, 2005).

More recently Silling (2000) introduced a continuum theory, the peridynamics (PD), which is capable of addressing problems involving discontinuities and/or long range forces. One of the main features of this theory is the representation of the equations of motion through integro-differential equations instead of PDEs. This relaxes, to a significant extent, the smoothness requirement of the deformation field and even allows for discontinuities as long as the Riemann integrability of the spatial integrals is ensured. These equations are based on a model of internal forces that the material points exert on each other over finite distances. The initial model, the *bond-based* PD, treats the internal forces as a network of interacting pairs like springs. The maximum distance through which a material particle interacts with its neighbors via spring like interactions is denoted as the *horizon*. Such pair-wise forces however lead to an oversimplification of the model and in particular results in an effective Poisson's ratio of 1/4 for linear isotropic elastic materials. This limitation has been overcome through a more general model, the *state-based* PD (Silling et al., 2007). According to this theory, particles interact via bond forces that are no longer governed by a central potential independent of the behavior of other bonds; instead they are determined by the collective deformations of the bonds within the horizon of a material particle. This version of the PD theory is applicable over the entire permissible range of Poisson's ratio. Even though the PD has many attractive features, the scarcity of strictly PD-based material constitutive models tends to limit its applicability. This difficulty may however be bypassed using a constitutive correspondence framework (Silling et al., 2007), which enables the use of classical material models in a PD formulation.

In the present work, a novel proposal for a PD approach incorporating micropolar elasticity is set forth. A set of state-based equations of motion is derived for the micropolar continuum and the constitutive correspondence utilized to define the associated material model. Incorporation of additional physical information via the material length-scale parameters has been a primary motivation in the current development. Such an enhancement of the model is expected to emulate more closely the physical behavior of structures like nano-beams, nano-sheets, fracture characteristics of thin films, concrete structures etc. In this context, an earlier work by Gerstle et al. (2007) on bond-based micropolar PD should be mentioned, which, whilst eliminating the issue of fixed Poisson's ratio, does not offer a ready framework to incorporate the rich repertoire of classical material models. The last work also has additional limitations in imposing the incompressibility constraint, often employed in a wide range of models including those involving plastic deformation in metals (Silling et al., 2007). Along with a general three dimensional model, a one dimensional micropolar PD model for a Timoshenko type beam is also derived in this work through an appropriate dimensional descent. For the purpose of comparison, a similar beam model based on the standard non-polar PD is derived. Effects of the length scale parameters on the static deformation characteristics of a beam under different boundary conditions are numerically assessed confirming the superiority of the (new) micropolar model over the non-polar one. A couple of two dimensional planar problems, first of a plate with a hole and the other involving a plate with a central crack under tensile loading, also hold out similar observations. The theoretical development in this article is, however, limited to linear isotropic elastic deformations only.

The rest of the paper is organized as follows. Section 2 briefly describes the state based PD theory and also gives a short account of linear elastic micropolar theory. While Section 3 reports on a systematic derivation of a general 3D micropolar PD theory, the one dimensional adaptations of the theory are laid out in Sections 4 and 5. This is followed by numerical illustrations and a few concluding remarks in Sections 6 and 7 respectively.

2. State based PD and micropolar elasticity

For completeness, a concise description of the state based PD theory along with the constitutive correspondence is given in this section. Equations of motion in the micropolar theory and the linear elastic material model are also briefly reviewed.

2.1. State based PD

Following the approach in Silling et al. (2007), a brief account of the state-based PD theory is presented below. PD is a non-local continuum theory that describes the dynamics of a body occupying a region $\mathcal{B}_0 \subset \mathbb{R}^3$ in its reference configuration and $\mathcal{B}_0 \subset \mathbb{R}^3$ in the current configuration. A schematic of the body is shown in Fig. 1. The *bond* vector ξ between a material point $\mathrm{X} \in \mathcal{B}_0$ and its neighbor $\mathrm{X}' \in \mathcal{B}_0$, defined as $\xi = \mathrm{X}' - \mathrm{X}$, gets deformed under the deformation map $\chi : \mathcal{B}_0 \rightarrow \mathcal{B}_t$. The deformed bond is given by the deformation vector state $\underline{\mathbf{Y}}$ (refer to Silling et al. (2007) for a precise definition of states)

$$\underline{\mathbf{Y}}[\mathrm{X}]\langle\xi\rangle = \mathrm{y}' - \mathrm{y} = \chi(\mathrm{X}') - \chi(\mathrm{X}) \qquad (1)$$

The family of bonds to be considered for a point X is given by its *horizon* $\mathcal{H}$ defined as $\mathcal{H}(\mathrm{X}) = \{\xi \in \mathcal{B}^3 | (\xi + \mathrm{X}) \in \mathcal{B}_0, |\xi| < \delta\}$, where $\delta > 0$ is the radius of the horizon.

The state based PD equations of motion are of the following integro-differential form.

$$\rho(\mathrm{X})\ddot{\mathrm{y}}(\mathrm{X},\mathrm{t}) = \int_{\mathcal{H}(\mathrm{X})} \{\underline{\mathbf{T}}[\mathrm{X},\mathrm{t}]\langle\xi\rangle - \underline{\mathbf{T}}[\mathrm{X}+\xi,\mathrm{t}]\langle-\xi\rangle\} dV_{\mathrm{X}'} + b(\mathrm{X},\mathrm{t}) \qquad (2)$$

where ρ, $\underline{\mathbf{T}}$, b are the mass density, long range internal force vector state and externally applied body force density respectively. Superimposed dots indicate material derivatives with respect to time. This equation has been shown in Silling et al. (2007) to satisfy the linear momentum balance. In the standard non-polar PD theory, conservation of angular momentum is ensured by imposing the following restriction on the constitutive relation.

$$\int_{\mathcal{H}(\mathrm{X})} \underline{\mathbf{T}}[\mathrm{X},\mathrm{t}]\langle\xi\rangle \times \underline{\mathbf{Y}}[\mathrm{X},\mathrm{t}]\langle\xi\rangle dV_{\mathrm{X}'} = 0, \quad \forall \mathrm{X} \in \mathcal{B}_0 \qquad (3)$$

Silling et al. (2007) have proposed a constitutive correspondence in order to incorporate the classical material models within the PD

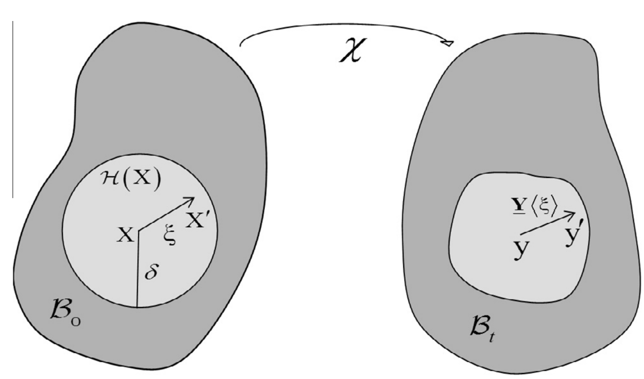

Fig. 1. Schematic PD body in the reference and current configurations.

Please cite this article in press as: Roy Chowdhury, S., et al. A micropolar peridynamic theory in linear elasticity. Int. J. Solids Struct. (2015), http://dx.doi.org/10.1016/j.ijsolstr.2015.01.018