International Journal of Solids and Structures 58 (2015) 12–19

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Radial wrinkles on film–substrate system induced by local prestretch: A theoretical analysis

Xiao Huang [a], Hong-Ping Zhao [a], Wei-Hua Xie [a], Wei Hong [b], Xi-Qiao Feng [a,c,*]

[a] Institute of Biomechanics and Medical Engineering, AML, Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China
[b] Department of Aerospace Engineering, Iowa State University, Ames, IA 50011, USA
[c] Center for Nano and Micro Mechanics, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 26 May 2014
Received in revised form 22 November 2014
Available online 31 December 2014

Keywords:
Radial wrinkle
Film–substrate structure
Hankel transform
Theoretical model

ABSTRACT

Local wrinkles are widely observed in film–substrate systems both in nature and in engineering. In this paper, we investigate the surface wrinkling of a film–substrate structure subjected to local prestretch in a circular region. Hankel transform is used to unravel the interface condition between the stiff film and the compliant substrate. The critical prestrain and the corresponding wrinkling number are solved as functions of the radius of the prestretched region and the Young's modulus ratio between the film and substrate. The theoretical analysis is validated by a semi-implicit numerical method based on the Fourier spectral technique. The postbuckling behavior is also simulated via the numerical method. It is found that during postbuckling, the surface wrinkles may experience a secondary bifurcation and evolve into branching patterns.

1. Introduction

The issue of surface wrinkling has attracted much attention in the past decades because of its technological significance in, for instance, stretchable or flexible electronic devices (Kim and Rogers, 2008), film metrology (Stafford et al., 2004), surface topography (Cai et al., 2011), medical engineering (Li et al., 2012), and colloidal crystal assemblies (Efimenko et al., 2005). Besides, surface wrinkling frequently occurs in biological tissues and systems with normal and morbid physiological processes, e.g., scar contraction (Cerda, 2005; Flynn and McCormack, 2008), skin corrugation (Genzer and Groenewold, 2006), mucosal morphogenesis (e.g., esophagus, airway, and stomach) (Li et al., 2011; Moulton and Goriely, 2011), and fruit shrinkage (Yin et al., 2008). For a stiff film lying on a compliant substrate subjected to compressive strain beyond a critical value, the system may buckle into various surface patterns, e.g., stripes, checkerboards, hexagons, herringbones, etc. (Cai et al., 2011; Cao et al., 2012; Chen and Hutchinson, 2004; Cheng et al., 2014; Huang et al., 2005; Li et al., 2010; Song et al., 2008). The wrinkling mode in a specific system depends on its geometric and material parameters, external loads, and boundary conditions.

In recent years, much effort has been directed toward understanding various wrinkling phenomena in freestanding thin films and film–substrate systems, including surface wrinkling induced by local loads or differential volume growth/shrinkage (Cerda, 2005; Chung et al., 2009; Coman and Bassom, 2007; Coman and Liu, 2013; Flynn and McCormack, 2008; Géminard et al., 2004; Qiao et al., 2013). Take eye wrinkles as an example. Human skin is comprised of a thin epidermis resting on top of a thick and much softer dermis layer (Genzer and Groenewold, 2006) and, therefore, it can be modeled as a stiff film–compliant substrate system. The appearance of eye wrinkles with aging is primarily due to the differential volumetric growth/shrinkage and elastic modulus variations of epidermis and dermis. Among various types of surface patterns, radial wrinkles have recently attracted much attention. Géminard et al. (2004) conducted an experiment in which the center of an elastic disc is sucked into a small ring, generating radial wrinkles around the inner ring when the suction is beyond a critical value. On the basis of experiments, Cerda (2005) established a scaling model to investigate the local wrinkling of skin due to scar contraction, with the influence of underlying supporting dermis neglected. Coman and Bassom (2007) performed numerical simulations and asymptotic analysis to study the radial wrinkling of an annular thin film without substrate in tension. They elucidated the exponential decay of wrinkles by using the Wentzel–Kramer–Brillouin theory (Coman and Haughton, 2006). Wang et al. (2014) studied, both experimentally and theoretically, the local surface wrinkling of a thin film on a compliant substrate subjected to surface torsion. They

* Corresponding author at: Institute of Biomechanics and Medical Engineering, AML, Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China. Tel.: +86 10 62772934; fax: +86 10 62781824.
E-mail address: fengxq@tsinghua.edu.cn (X.-Q. Feng).

http://dx.doi.org/10.1016/j.ijsolstr.2014.12.011
0020-7683/

found that each spiral wrinkle has the shape of an Archimedean spiral curve. Besides, an elastic sheet floating on a fluid may buckle due to the capillary force at the solid–liquid contact line, referred to as elastocapillary buckling (Huang et al., 2007; Liu and Feng, 2012; King et al., 2012; Toga et al., 2013; Vella et al., 2010).

Radial wrinkles often occur in biological tissues and engineering structures. However, the critical condition for the occurrence of radial wrinkles and their morphological evolution of film–substrate systems remains unclear. In this paper, therefore, we will investigate, through theoretical analysis and numerical simulations, the local radial wrinkling of a thin stiff film lying on an infinite compliant substrate engendered by a prestretch in a circular region. The layout of this paper is as follows. In Section 2, we develop a theoretical model to analyze the critical radial buckling behavior of a film–substrate structure. Hankel transform technique is introduced to account for the effect of elastic interaction between the film and the substrate. In Section 3, a semi-implicit numerical algorithm is utilized to calculate the morphological evolution and validate the theoretical model. The results are given in Section 4, where two important features of the normalized radius of the prestretched area and the film/substrate modulus ratio are discussed. Besides, a curve-fitting analysis is made to simplify the theoretical results and a postbuckling simulation is further conducted when the prestretch is beyond the critical value. Finally, Section 5 gives the conclusions drawn from this study.

2. Theoretical analysis

2.1. Model

Consider a stiff thin film of thickness h, perfectly bonded to a compliant semi-infinite elastic substrate. In a circular area of radius r_0, the film has a homogeneous prestretch with equal-biaxial strain ε_0. This model can mimic such film–substrate systems as skin with local strain induced by different reasons, e.g., differential growth in scar. Refer to a cylindrical coordinate system $(r,\ \theta,\ z)$, as shown in Fig. 1, where the origin O is located at the center of the prestretched region, and the r and θ axes are along and normal to the radial direction, respectively. Both the film and the substrate are assumed to be linear elastic and isotropic, with Young's moduli E_f and E_s, and Poisson's ratios ν_f and ν_s, respectively. When the prestrain ε_0 reaches a critical value, the region outside the prestretched area may wrinkle into a radial or spoke-like morphology because the maximal compressive stress is along the circumferential direction. The unbuckled state of the system under the given prestrain ε_0 is axisymmetric and taken as the reference configuration. The corresponding displacements in the film outside the prestretched area $(r \geqslant r_0)$ are

$$\bar{u}_r^\mathrm{f} = -\frac{\varepsilon_0(1+\nu_\mathrm{f})r_0^2}{2r}, \quad \bar{u}_\theta^\mathrm{f} = \bar{w}^\mathrm{f} = 0, \tag{1}$$

where $\bar{u}_r^\mathrm{f}$, $\bar{u}_\theta^\mathrm{f}$, and $\bar{w}^\mathrm{f}$ denote the radial, circumferential, and out-of-plane displacements, respectively. The corresponding strains and stresses can be written as

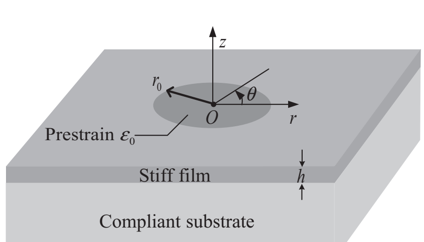

Fig. 1. A stiff film resting on a compliant substrate. The film has a uniform biaxial prestretch in the circular area.

$$\bar{\varepsilon}_{rr}^\mathrm{f} = \frac{\varepsilon_0(1+\nu_\mathrm{f})}{2}\left(\frac{r_0}{r}\right)^2, \quad \bar{\varepsilon}_{\theta\theta}^\mathrm{f} = -\frac{\varepsilon_0(1+\nu_\mathrm{f})}{2}\left(\frac{r_0}{r}\right)^2, \quad \bar{\varepsilon}_{r\theta}^\mathrm{f} = 0, \tag{2}$$

$$\bar{\sigma}_{rr}^\mathrm{f} = \frac{E_\mathrm{f}\varepsilon_0}{2}\left(\frac{r_0}{r}\right)^2, \quad \bar{\sigma}_{\theta\theta}^\mathrm{f} = -\frac{E_\mathrm{f}\varepsilon_0}{2}\left(\frac{r_0}{r}\right)^2, \quad \bar{\sigma}_{r\theta}^\mathrm{f} = 0. \tag{3}$$

When the prestrain exceeds a critical value, the film would buckle into an actinomorphic morphology. Here it is assumed that the wrinkling wavelength is much larger than the amplitude, so that the von Kármán nonlinear elastic plate theory can be applied to describe the deformation of the film (Landau and Lifshitz, 1959). Let w^f denote its deflection in the z direction, u_r^f and u_θ^f the in-plane displacements in the r and θ directions, respectively. Then the total displacements can be decomposed as $u_r^\mathrm{f} = \bar{u}_r^\mathrm{f} + u_r^\mathrm{b}$, $u_\theta^\mathrm{f} = \bar{u}_\theta^\mathrm{f} + u_\theta^\mathrm{b}$, and $w^\mathrm{f} = \bar{w}^\mathrm{f} + w^\mathrm{b}$, where u_r^b, u_θ^b, and w^b represent the additional displacement due to buckling in the r, θ, and z directions, respectively. In the polar coordinate system (r, θ), the strains $\varepsilon_{\alpha\beta}^\mathrm{f}$ in the film can be expressed as

$$\begin{aligned}
\varepsilon_{rr}^\mathrm{f} &= \frac{\partial u_r^\mathrm{f}}{\partial r} + \frac{1}{2}\left(\frac{\partial w^\mathrm{f}}{\partial r}\right)^2, \\
\varepsilon_{\theta\theta}^\mathrm{f} &= \frac{1}{r}\frac{\partial u_\theta^\mathrm{f}}{\partial \theta} + \frac{u_r^\mathrm{f}}{r} + \frac{1}{2r^2}\left(\frac{\partial w^\mathrm{f}}{\partial \theta}\right)^2, \\
\varepsilon_{r\theta}^\mathrm{f} &= \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r^\mathrm{f}}{\partial \theta} + \frac{\partial u_\theta^\mathrm{f}}{\partial r} - \frac{u_\theta^\mathrm{f}}{r} + \frac{1}{r}\frac{\partial w^\mathrm{f}}{\partial r}\frac{\partial w^\mathrm{f}}{\partial \theta}\right).
\end{aligned} \tag{4}$$

Using Hooke's law and Eq. (4), the membrane forces in the film can be obtained as

$$N_{\alpha\beta} = \bar{E}_\mathrm{f} h\left[(1-\nu_\mathrm{f})\varepsilon_{\alpha\beta}^\mathrm{f} + \nu_\mathrm{f}\varepsilon_{\gamma\gamma}^\mathrm{f}\delta_{\alpha\beta}\right], \tag{5}$$

where $\bar{E}_\mathrm{f} = E_\mathrm{f}/(1-\nu_\mathrm{f}^2)$ is the plane-strain elastic modulus of the film and $\delta_{\alpha\beta}$ is the Kronecker delta. Einstein summation convention is adopted for repeated Greek indices, which take the values of 1 (or r) and 2 (or θ).

2.2. Three-dimensional solution of film–substrate interaction at the radial wrinkling state

The interaction between the thin film and the substrate plays a significant role in the wrinkling of the system. In this study, the effect of shear stresses at their interface will be taken into account. However, this problem is mathematically complicated, involving three-dimensional and non-axisymmetric wrinkling deformations of the film and the substrate. In what follows, we introduce the Hankel transform to solve this problem.

Let T_r and T_θ denote the shear stresses and T_z denote the normal stress at the film–substrate interface. Then the equilibrium equations of the film are expressed as

$$\begin{aligned}
T_r &= \frac{\partial N_{rr}}{\partial r} + \frac{1}{r}\frac{\partial N_{r\theta}}{\partial \theta} + \frac{1}{r}(N_{rr} - N_{\theta\theta}), \\
T_\theta &= \frac{\partial N_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial N_{\theta\theta}}{\partial \theta} + \frac{2}{r}N_{r\theta}, \\
T_z &= -\frac{h^3\bar{E}_\mathrm{f}}{12}\nabla^4 w^\mathrm{f} + T_r\frac{\partial w^\mathrm{f}}{\partial r} + T_\theta\frac{1}{r}\frac{\partial w^\mathrm{f}}{\partial \theta} + N_{rr}\frac{\partial^2 w^\mathrm{f}}{\partial r^2} \\
&\quad + 2N_{r\theta}\left(\frac{1}{r}\frac{\partial^2 w^\mathrm{f}}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial w^\mathrm{f}}{\partial \theta}\right) + N_{\theta\theta}\left(\frac{1}{r^2}\frac{\partial^2 w^\mathrm{f}}{\partial \theta^2} + \frac{1}{r}\frac{\partial w^\mathrm{f}}{\partial r}\right),
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
\nabla^4 w^\mathrm{f} &= \frac{\partial^4 w^\mathrm{f}}{\partial r^4} + \frac{2}{r}\frac{\partial^3 w^\mathrm{f}}{\partial r^3} - \frac{1}{r^2}\left(\frac{\partial^2 w^\mathrm{f}}{\partial r^2} - 2\frac{\partial^4 w^\mathrm{f}}{\partial r^2\partial\theta^2}\right) \\
&\quad + \frac{1}{r^3}\left(\frac{\partial w^\mathrm{f}}{\partial r} - 2\frac{\partial^3 w^\mathrm{f}}{\partial r\partial\theta^2}\right) + \frac{1}{r^4}\left(4\frac{\partial^2 w^\mathrm{f}}{\partial \theta^2} + \frac{\partial^4 w^\mathrm{f}}{\partial \theta^4}\right)
\end{aligned} \tag{7}$$