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Influence of internal impacts between fragments in dynamic brittle tensile fragmentation

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ABSTRACT

Dynamic fragmentation phenomena involve two important mechanisms: atomic debonding and internal impacts among formed fragments. While the former, i.e. energy dissipation due to crack propagation, has been frequently studied both theoretically and numerically, the same does not apply to the latter due to the inherent difficulty of tracking contact occurrences. In fact, in order to simplify computations, fragment to fragment interactions are often neglected when dealing with tensile expansion. We study the validity of this assumption using a simple 1D finite-element model. We consider a brittle bar subjected to tensile loading, and model material failure with cohesive elements. We show that neglecting internal contacts has little influence at high loading rates. However, our results reveal that even in initially pure tensile cases, for low strain rates and brittle materials, fragment interactions drastically change the fragmentation process as well as the fragments' residual velocities.

1. Introduction

Dynamic fragmentation occurs when a material is subjected to extreme loading. The failure is rapid and catastrophic. Cracks initiate at internal material defects, propagate at high speeds and coalesce to form fragments, which can move and impact each other. The process is complex: it is fast transient, involves several dissipating mechanisms, and is characterized by a high unpredictability. Some aspects, such as fragments sizes distribution resulting from dynamic fragmentation, have been extensively studied while others, such as fragments residual velocities distribution have received less attention. Noteworthy exceptions include the astrophysics and defense communities (see for example Singer et al., 2013; Grady, 1999; Zukas, 1982). Initially research on this topic was carried out experimentally and analytically. The first notable experiments were accomplished by Rosin and Rammler (1933) and Mott and Linfoot (1943), together with the theoretical work of Lienau (1936). Some semi-empirical models correlating masses and velocities were proposed more recently. In particular for the mass-velocity relation the following power law was introduced by Nakamura and Fujiwara (1991)

$$v \propto m^{-k} \tag{1}$$

where v and m are the fragment's velocity and mass, while k is an experimental constant that in high-velocity impact (km/s) regime was found to be $k \approx 1/6$ for basalt and alumina. However, following studies conducted by Giblin (1998) on an artificial rock similar to basalt, showed that for this specific case a power law did not apply. A similar behavior was observed by Kadono et al. (2005) on thin glass plates. Also the defense community developed some formulas to estimate the velocity of fragments. An example is represented by the Gurney's equation, namely a power law relating velocity and mass (Gurney, 1943). For a fragment released in the explosion of a cylindrical shell, this equation states that

$$v = \sqrt{2E}\left(\frac{M}{C}+\frac{1}{2}\right)^{-\frac{1}{2}} \tag{2}$$

where v is the fragment velocity in m/s, M the mass of the casing, C the mass of the explosive (same unit as M), and $\sqrt{2E}$ is called Gurney constant and its unit is m/s.

Thanks to the considerable and fast development of computers, numerical methods have become an increasingly effective tool for researchers. As far as dynamic fragmentation is concerned, two main numerical approaches can be distinguished: particle and continuum methods. In the first case materials are represented by a finite number of discrete particles. This approach permits to handle discontinuities and big deformations more easily, but at the same time usually leads to costly computations and adds a non-physical length scale due to spacing among particles (it is only at the atomic scale that materials can effectively be represented by particles).

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A classical example is the smooth particle hydrodynamics (SPH), that was born in the late '70s (Gingold and Monaghan, 1977; Lucy, 1977) and was employed in solid mechanics impact simulations for the first time by Libersky et al. (1993). According to the second approach, materials are initially represented as continuum bodies in order to capture rigorously the mechanical behavior together with a low computational cost. The drawback is that modeling discontinuities (cracks) is challenging. Several approaches can be selected. In the continuum-damage approach the elements' stiffness is progressively decreased along crack paths. The nonlocal formulation (see for example Pijaudier-Cabot and Bažant, 1987) proved to be an effective mesh independent method for fracture mechanics. However it does not include contact and crack paths are not precisely defined but spread over several elements. An alternative method consists in using cohesive elements, which are zero thickness elements that let cracks propagate along standard elements' edges. They have been successfully utilized to model fragmentation upon impact by Camacho and Ortiz (1996). The limitation of this method is its mesh dependency, since cracks are forced to pass through elemental edges. Mesh dependency and convergence have been analyzed by Zhou and Molinari (2004), Molinari et al. (2007) and Levy and Molinari (2010). The eXtended Finite Element Method (XFEM) has been developed by Moës et al. (1999) to obtain mesh independent crack paths. The method is however not adequate for fragmentation, which involves extensive crack branching and coalescence patterns.

No matter which numerical method is used, the main objective of realistic dynamic fragmentation simulations is to account for all important mechanisms including failure and internal contacts. While much attention has been given to reproducing an accurate crack network, in tensile fragmentation simulations the influence of contact has often been neglected (e.g. Levy et al., 2012). Therefore the aim of this article is to analyze the importance of contact on such simulations. The following data will be tracked:

1. number of fragments;
2. dissipated energy;
3. residual velocities.

For this purpose a very simple case is considered: a brittle quasi-1D bar under uniform traction modeled through FEM with cohesive elements. In this problem, when cracking occurs, stress waves propagate inside the material and consequently fragments repeatedly shrink and expand, generating multiple contacts among themselves. First, a reference case without energy dissipation and contact is analyzed. Then, these last two physical components are sequentially added.

Section 2 contains a description of the model, from its underlying equations to the boundary and initial conditions, etc. In Section 3 the results obtained when contact is neglected are presented. The fragments' velocities are monitored throughout the simulation and compared to the initial ones. Finally Section 4 shows results obtained after having introduced contact in the model. Also in this case the fragments' velocities are compared with the initial ones and the differences with respect to the case without contact are highlighted.

2. Numerical setup

2.1. Mechanical model and cohesive rupture

We solve the equation of motion

$$\nabla \cdot \boldsymbol{\sigma} = \rho\, \ddot{\mathbf{u}} \tag{3}$$

where $\boldsymbol{\sigma}$ is the stress tensor, ρ is the density and $\ddot{\mathbf{u}}$ is the double derivative of displacement with respect to time, namely acceleration. No body force is considered in this work. The solver is based on two levels of discretization: over space (with elements) and over time. Integration over time is carried out with an explicit central difference integration scheme. Further details can be found for example in Belytschko et al. (2000).

In FEM cracks implementation is often based on the cohesive zone concept introduced by Dugdale (1960) and Barenblatt (1962). The relationship between surface traction and crack opening displacement is called cohesive law. In this work the linear irreversible cohesive law proposed by Camacho and Ortiz (1996) is used (see Fig. 1). However, in our quasi-1D application, an advanced management of contact is not necessary, because contact surfaces are immediately determined and no sliding occurs. Therefore contact can be integrated in the cohesive law by means of a penalty coefficient α. The cohesive tractions T per unit deformed area for a given opening δ are

$$T(\delta, \delta_{\max}) = \begin{cases} \sigma_c\left(1 - \frac{\delta}{\delta_c}\right) & \text{for } \delta = \delta_{\max} \\ \frac{T_{\max}}{\delta_{\max}}\,\delta & \text{for } 0 \leqslant \delta < \delta_{\max} \\ \alpha\,\delta & \text{for } \delta < 0 \end{cases} \tag{4}$$

in which three cases are taken into account:

- crack opening;
- crack closure/reopening;
- interpenetration.

At every time step $\delta_{\max}$ is updated. After reaching $\delta_{\max} = \delta_c$, any tensile traction is zero but compressive tractions may still occur at a later stage if the crack faces are forced to close. In our model, the dissipated and reversible energies can be defined as

$$E_{\text{diss}} = \frac{1}{2}\sigma_c\,\delta_{\max} \tag{5}$$

$$E_{\text{rev}} = \frac{1}{2}T\,\delta \tag{6}$$

and so when decohesion is complete $E_{\text{diss}} = G_c = \frac{1}{2}\sigma_c\,\delta_c$. Moreover a damage parameter D can be defined as

$$D = \min\left(\frac{\delta_{\max}}{\delta_c}, 1\right) \tag{7}$$

which varies from 0 (undamaged condition) to 1 (fully damaged condition). This variable can only increase because damage is an

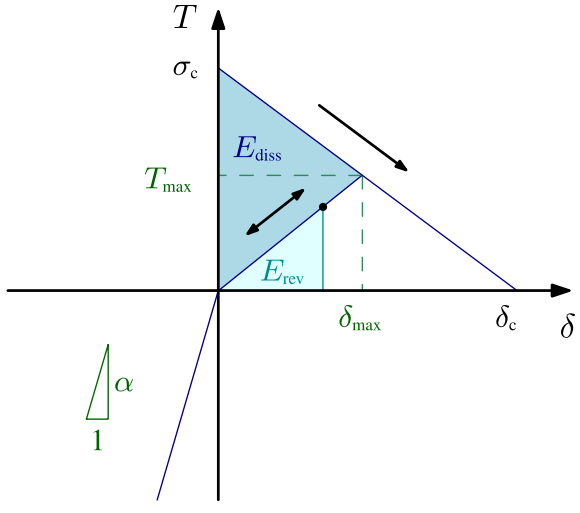

Fig. 1. Linear decreasing cohesive law.