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Buckling and postbuckling of variable angle tow composite plates under in-plane shear loading

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ABSTRACT

A geometrically nonlinear analysis of symmetric variable angle tow (VAT) composite plates under in-plane shear is investigated. The nonlinear von Karman governing differential equations are derived for postbuckling analysis of symmetric VAT plate structures which are subsequently solved using the differential quadrature method. The effect of in-plane extension-shear coupling on the buckling and postbuckling performance of VAT composite plates is investigated. The buckling and postbuckling behaviour of VAT plates under positive and negative shear is studied for different VAT fibre orientations, aspect ratios, combined axial compression and their performance is compared with that of straight fibre composites. It is shown that there can be enhanced shear buckling and postbuckling performance for both displacement-control and load-control and that the underpinning driving mechanics are different for each.

1. Introduction

Stability analysis of variable angle tow (VAT) composites under compression load has been studied extensively and response has been shown to have superior structural performance over conventional straight fibre composites (Hyer and Lee, 1991; Gurdal and Olmedo, 1993; Gurdal et al., 2008). In this work, the buckling and postbuckling behaviour of VAT plates under in-plane shear is investigated. The concept of tow steering provides more freedom to design light-weight composite structures with improved structural performance when compared to traditional straight fibre designs. Little work has been reported on the stability analysis of VAT plates under shear load. Biggers and Fageau (1994) studied the concept of stiffness tailoring for improving the shear buckling performance of composite plates by redistributing the layups with certain fibre orientations across the planform of the plate. Their study showed a 50% improvement of shear buckling load over straight fibre composites by redistributing ±45 plies along the diagonal directions. Waldhart (1996) used the Rayleigh–Ritz method to study the buckling performance of tow steered VAT plates under uniform end-shortening and in-plane shear load. The effects of extensional-shear coupling (A_{16}, A_{26}) were not considered in their shear buckling study. Nemeth (1997) performed a parametric study on the buckling behaviour of long symmetrical composite plates under shear and reported the effects of membrane anisotropy are more important for shear loaded plates than compression or in-plane bending. Weaver (2004) studied the elastic tailoring of long composite laminates using both flexural and membrane anisotropy and quantified their effects on positive/negative shear buckling behaviour. Wu et al. (2012) studied the buckling performance of VAT plates under compression, shear and combined loading using energy methods. Their study investigated the effect of extensional-shear coupling (A_{16}, A_{26}) and bend-twist coupling (D_{16}, D_{26}) on the buckling behaviour of VAT plates. Lopes et al. (2010) and Gomes et al. (2013) studied the buckling and postbuckling failure response of variable stiffness composites with cut-outs under compression and shear loading, respectively. They used finite element analysis to model the failure of VAT plates which requires significant computational effort. Rahman et al. (2011) studied the postbuckling response of VAT plates using a perturbation approach, coupled with finite element modelling, to generate a reduced-order model for computation of postbuckling coefficients to predict the postbuckling stiffness of VAT plates. Wu et al. (2013) studied the postbuckling performance of VAT plates under axial compression with linear fibre angle variation for different in-plane boundary conditions and proposed different measures to quantify the postbuckling performance of VAT plates.

Numerous studies on VAT plates rely on finite element (FE) modelling for analysis and design of these structures. As a consequence of variable stiffness coefficients, prebuckling stress distributions can be highly nonlinear (spatially) in-plane, even for

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uniform loading, and not immediately intuitive (Gurdal and Olmedo, 1993V). Furthermore, it is not obvious whether an FE mesh which is converged for prebuckling analysis will also be converged for buckling or for subsequent postbuckling analyses. A further limitation of FE analysis is that rather than modelling continuous fibre paths, the fibre angle distribution is treated as piecewise constant within each element, leading to spurious stress and strain residuals (i.e. noise) in coarse meshes. Therefore, a strong need for developing semi-analytical models that complements finite element analysis for modelling of VAT panel is required. In the present work, numerical methodology based on the differential quadrature method (DQM) is developed for buckling and postbuckling analysis of VAT panels under in-plane shear load. In prior works, the authors have successfully applied DQM for evaluation of buckling and postbuckling behaviour of VAT plates under compression for different plate boundary conditions (Raju et al., 2012, 2013). DQM, as a numerical tool, has been shown to be accurate and require less degrees of freedom than FE for solving the buckling and postbuckling problem of VAT panels. Once simple geometries in FE analysis have been validated by DQM models, then the designer can proceed with increased confidence to more complicated geometries and loads. More importantly still, is the physical insight gained in stress redistribution tailoring, and the ability to massage buckling phenomena to be more benign.

In the present work, the underlying mechanics behind the improvement of shear buckling and postbuckling behaviour of VAT plates with linear fibre angle variation is studied. The effect of in-plane extension-shear coupling on the buckling and postbuckling performance of VAT composite plates is investigated for different in-plane boundary conditions. Furthermore, the effect of direction of the applied shear on the postbuckling behaviour of VAT plates under compression is also discussed.

2. Differential quadrature method

In the differential quadrature method, the derivative of a function, with respect to a space variable at a given discrete grid point, is approximated as a weighted linear sum of the function values at all of the grid points in the entire domain of that variable (Bellman and Casti, 1971). The nth order partial derivative of a function $f(x)$ at the ith discrete point is approximated by

$$\frac{\partial^n f(x_i)}{\partial x^n} = A_{ij}^{(n)} f(x_j) \quad i = 1, 2, \ldots, N_x, \tag{1}$$

where x_i = set of discrete points in the x direction; and $A_{ij}^{(n)}$ are the weighting coefficients of the nth derivative and repeated index j indicates summation from 1 to N_x. The partial derivatives of a function $f(x,y)$ in matrix form are given by,

$$\begin{aligned}
&\frac{\partial f}{\partial x} = P_x f, \quad \frac{\partial f}{\partial y} = f P_y^T, \quad \frac{\partial f}{\partial x \partial y} = P_x f P_y^T,\\
&\frac{\partial^2 f}{\partial x^2} = Q_x f, \quad \frac{\partial^2 f}{\partial y^2} = f Q_y^T, \quad \frac{\partial^4 f}{\partial x^2 \partial y^2} = Q_x f Q_y^T,\\
&\frac{\partial^3 f}{\partial x^3} = R_x f, \quad \frac{\partial^3 f}{\partial y^3} = f R_y^T, \quad \frac{\partial^4 f}{\partial x^4} = S_x f, \quad \frac{\partial^4 f}{\partial y^4} = f S_y^T,
\end{aligned} \tag{2}$$

where $P,\ Q,\ R,\ S$ with subscripts $x,\ y$ are the DQM weighting coefficient matrices for the first, second, third, and fourth order partial derivatives with respect to x and y directions, respectively. The unknown function f is expressed in matrix form along the two-dimensional grid, as shown in Fig. 1 and superscript T represents the transpose of the matrix. The domain grid points refer to the points where the governing partial differential equations are expressed in DQM form and the boundary grid points refer to the points where multiple boundary conditions are applied (Fig. 1). The information regarding the grid distribution for computation of weighting coefficient matrices and modelling multiple boundary conditions are explained, in detail, in the textbook by Shu (2000).

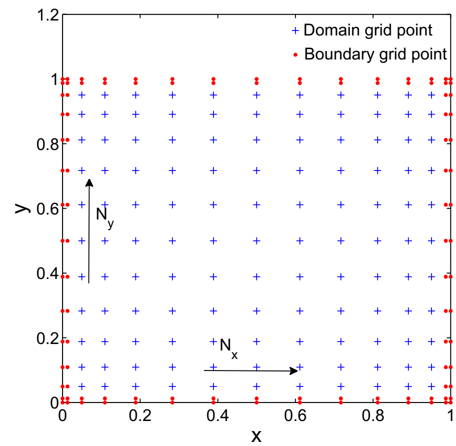

Fig. 1. DQM grid distribution in two-dimension.

3. Postbuckling analysis of VAT panels

In symmetric VAT panels, stiffness (A, D matrices) varies with x–y coordinates and the constitutive equation in partial inverse form is given by,

$$\begin{Bmatrix} \epsilon^0 \\ M \end{Bmatrix} = \begin{bmatrix} A^*(x,y) & 0 \\ 0 & D(x,y) \end{bmatrix} \begin{Bmatrix} \overline{N} \\ \kappa \end{Bmatrix}, \tag{3}$$

where $\overline{N},\ M$ are the stress and moment resultants, $A^* = A^{-1}$ is the compliance matrix and D is the bending stiffness matrix. The nonlinear midplane strains ϵ^0 and curvatures κ are defined as

$$\begin{aligned}
&\epsilon_x^0 = u_{,x} + \frac{1}{2} w_{,x}^2 + w_{,x} w_{0,x}, \quad \epsilon_y^0 = v_{,y} + \frac{1}{2} w_{,y}^2 + w_{,y} w_{0,y},\\
&\epsilon_{xy}^0 = u_{,y} + v_{,x} + w_{,x} w_{,y} + w_{,x} w_{0,y} + w_{,y} w_{0,x},\\
&\kappa_x = -w_{,xx}, \quad \kappa_y = -w_{,yy}, \quad \kappa_{xy} = -2 w_{,xy}
\end{aligned} \tag{4}$$

where $u,\ v,\ w$ are the displacements and w_0 is the initial imperfection function. A stress function Ω is introduced such that the stress resultants are defined by,

$$\overline{N}_x = \Omega_{,yy}, \quad \overline{N}_y = \Omega_{,xx}, \quad \overline{N}_{xy} = -\Omega_{,xy}. \tag{5}$$

The compatibility condition in terms of mid-plane strains in a plane stress condition is given by (Whitney, 1987)

$$\begin{aligned}
\epsilon_{x,yy}^0 + \epsilon_{y,xx}^0 - \epsilon_{xy,xy}^0 = {} & w_{,xy}^2 - w_{,xx} w_{,yy} + 2 w_{,xy} w_{0,xy} - w_{,xx} w_{0,xx}\\
& - w_{,yy} w_{0,xx}.
\end{aligned} \tag{6}$$

After substitution of Eqs. (3)–(5) into Eq. (6), the final form is given by

$$\begin{aligned}
& A_{11}^*(x,y)\Omega_{,yyyy} - 2A_{16}^*(x,y)\Omega_{,xyyy} + (2A_{12}^*(x,y) + A_{66}^*(x,y))\Omega_{,xxyy}\\
& - 2A_{26}^*(x,y)\Omega_{,xxxy} + A_{22}^*(x,y)\Omega_{,xxxx} + (2A_{11,y}^*(x,y)\\
& - A_{16,x}^*(x,y))\Omega_{,yyy} + (2A_{12,x}^*(x,y) - 3A_{16,y}^*(x,y)\\
& + A_{66,x}^*(x,y))\Omega_{,xyy} + (2A_{12,y}^*(x,y) - 3A_{26,x}^*(x,y)\\
& + A_{66,y}^*(x,y))\Omega_{,xxy} + (2A_{22,x}^*(x,y) - A_{26,y}^*(x,y))\Omega_{,xxx}\\
& + (A_{11,yy}^*(x,y) + A_{12,xx}^*(x,y) - A_{16,xy}^*(x,y))\Omega_{,yy} + (-A_{26,xx}^*(x,y)\\
& - A_{16,yy}^*(x,y) + A_{66,xy}^*(x,y))\Omega_{,xy} + (A_{12,yy}^*(x,y) + A_{22,xx}^*(x,y)\\
& - A_{26,xy}^*(x,y))\Omega_{,xx}\\
& = w_{,xy}^2 - w_{,xx} w_{,yy} + 2 w_{,xy} w_{0,xy} - w_{,xx} w_{0,xx} - w_{,yy} w_{0,xx}.
\end{aligned} \tag{7}$$

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