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An extension of the linear stability analysis for the prediction of multiple necking during dynamic extension of round bar

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ABSTRACT

Linear stability analysis has been widely used in order to describe the evolution of the dominant necking pattern in different configurations. Effects of hardening, strain rate and temperature sensitivity, and effects of configuration geometry and loading, have been established by this mean. However, experimental and numerical observations have demonstrated that a whole distribution of spacing between necks is obtained instead of a unique dominant pattern. In this work, an extension of the classical linear stability analysis applied to the dynamic extension of a round bar case has been developed to take into account the contribution of all perturbation modes on the preliminary evolution of pre-necks. This approach, corresponding physically to the case of thin ring expansion, is able to determine a distribution of pre-neck spacing. This distribution, starting from the initial perturbation pattern, evolves with time so that it is finally centered around a dominant distance determined by the linear stability approach.

1. Introduction

The fragmentation of structures subject to dynamic conditions is a matter of interest in many applications concerning aerospace, automotive or Defence domains. A large number of studies were performed in the 40–60's to understand the explosion of bombs, warheads or casings (see for example Mott (1947)). Apart from Defence institutions which are still interested in a precise characterization of armament fragmentation, civil industries are also focusing on the hazardous effects of structure fragmentation. An example dedicated to the explosion of an acetylene gas cylinder, occurred in Sydney in 1993, can be seen in Price (2006).

Many documented experimental works have been conducted to investigate the fragmentation process on structures such as cylinders, rings, spheres and hemispherical shells, subject to explosive or electromagnetic loadings. The expansion and fragmentation of explosively driven structures have been, for example, investigated by Taylor (1963), Olive et al. (1979), Hoggatt and Recht (1968), Fyfe and Rajendran (1980) and Mock and Holt (1983) and recently by Hiroe et al. (2008) and Goto et al. (2008) for cylinders, by Hoggatt and Recht (1969), Goubot (1994) and Llorca and Juanicotena (1997) for rings and by Slate et al. (1967), Juanicotena (1998) and Mercier et al. (2010) for spherical and hemispherical shells. Electromagnetic loading is a second route to expand tubes and rings, see Niordson (1965), Wesenberg and Sagartz (1977), Grady and Benson (1983), Altynova et al. (1996), Grady and Olsen (2003) and Zhang and Ravi-Chandar (2006). Fragmentation of shaped charge jets has also been investigated intensively, see for example Karpp and Simon (1976), Chou et al. (1977) and Petit et al. (2005). From those experiments, information can be obtained on the fragment size distribution, the strain to necking or to failure, the number of necks or the onset time of instability or failure.

Numerous theoretical contributions were proposed to explain the fragmentation process or the development of instabilities observed in experiments. Some predictions of the distribution of fragment size are based on the work of Mott (1947) who developed a probabilistic theory associated to the concept of release wave originating at a fracture point. Revisiting this work, Grady (1981) has been able to retrieve the fragment distribution observed by Wesenberg and Sagartz (1977).

From experimental observations on expansion of rings (Zhang and Ravi-Chandar (2006)), the fragmentation can be viewed as a three stage process. First, the structure expands in a homogeneous way. In a second step, a multiple necking pattern occurs. Thirdly, some of the necks develop and cause failure, some of the necks are arrested. Therefore, predictions of the onset time for multiple

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necking and of the neck spacing are really fundamental to propose a clear view of the entire process, from homogeneous deformation to fracture. Stability and bifurcation analyses are widely used to tackle this problem. After Considère (1885), important developments were proposed by Hill and Hutchinson (1975) with a bifurcation analysis of a rectangular plate subject to plane strain traction for rate independent materials and by Hutchinson and Neale (1977) for rate dependent materials. These two fundamental works were done for quasi-static loadings. Later, Fressengeas and Molinari (1985, 1994) have extended Hutchinson and Neale (1977) contribution by incorporating inertia and the multidimensionality of the flow. They have shown that the interplay between inertia and effect of multidimensional flow and rate sensitivity leads to the selection of a dominant mode of perturbation with a defined wavelength. One can also mention the work of Shenoy and Freund (1999) who revisited previous results (Sorensen and Freund, 1998) by proposing an elegant formalism for the stability analysis of a plate. Mercier and Molinari (2003) used later this method to extend the works of Fressengeas and Molinari (1994) and Jeanclaude and Fressengeas (1997) and proposed a synthesis of the dynamic extension of plate and cylinder for rate independent and rate dependent materials. Mercier and Molinari (2004) analyzed the stability of the tube expansion. They observed that the curvature of the tube or ring may affect the necking pattern by changing the necking spacing. This effect may become significant when increasing the ratio thickness/mean radius. Nevertheless, when the tube is thin, a plate theory (neglecting the curvature) provides identical results. With linear stability analysis, effects of parameters like mass density, strain rate sensitivity, strain hardening and temperature softening coefficients or aspect ratio of the structure can be investigated, providing trends in agreement with experiments. Nevertheless, the main difficulty with stability analysis is the predictive capability in terms of time at onset of multiple necking. For example, Guduru et al. (2006) (and Dudzinski and Molinari (1991) in the quasi-static case) have proposed to define a criterion based on a critical value N_c of the ratio between the growth rate of the selected perturbation and the nominal strain rate of the background solution. With N_c around 14, the experimental results of Grady and Benson (1983) on Copper and Aluminum are retrieved. More recently, Jouve (2010) has adopted the same criterion for the analysis of cylinder expansions performed by Olive et al. (1979) on steel. With $N_c = 10$, the number of necks and the time at onset of multiple necking agree with the data of Olive et al. (1979). In addition, Jouve (2010) analyzed by numerical simulations, the extension of a plate where the perturbation adopted in the linear stability analysis is superimposed to the nominal geometry. He observed that the growth rates of the perturbation predicted by the linear stability analysis and the finite element calculations are consistent.

To understand the fragmentation process, numerous authors have made use of numerical simulations. Johnson (1981) simulated the fragmentation of an expanding ring using a one dimensional finite difference code. A Poisson distribution of porosity is introduced to trigger the instability process. He observed that the fragment size distribution depends on the initial porosity. Later, Han and Tvergaard (1995) proposed an interesting 2D numerical study for ring expansion. They introduced sinusoidal variation of the cross-section to initiate the process of multiple necking. They found that the number of necks in the ring is not linked to the initial perturbation introduced in the calculations when its magnitude is small. The amplitude of the imperfection accelerates the onset of necking. Works of Sorensen and Freund (2000) and Rodríguez-Martínez et al. (2013) on unimodal initial perturbations can also be mentioned. Many other simulations of expanding rings or cylinders have been performed in the recent years. To trigger instability, random geometrical or material imperfections can be introduced, see for instance Guduru and Freund (2002), Becker (2002), Zhou et al. (2006a), Zhang and Ravi-Chandar (2008), Petit (2010) and Hopson et al. (2011). For other authors, the numerical round-off can trigger the fragmentation, see Pandolfi et al. (1999), Becker (2002), Rusinek and Zaera (2007) and Meulbroek et al. (2008). In all those works, the fragment size is highly heterogeneous. The number of fragments at the end of the deformation is consistent with experiments. It can be noted as well that Putelat and Triantafyllidis (2014) have used similar numerical simulations to understand dynamic buckling of rings subjected to external hydrostatic pressure pulses and compared the results to a linear stability analysis.

In the present paper, we propose to analyze the dynamic traction of a round bar with initial geometrical defects. The cross-section of the bar is thus non-uniform with a bar profile along the longitudinal axis made of a succession of peaks and valleys. How this profile evolves with plastic deformation is the scope of the paper. It should be noted that the initial defects are assumed to be very small and will be treated as infinitesimal perturbation with respect to an otherwise uniform background bar cross-section. The configuration of the present paper is representative of expansion of rings with an initial surface roughness when the mean ring-radius is large compared to the dimensions of the cross-section. The evolution of perturbations is considered in the framework of a linearized perturbation method (linear stability analysis) for thermo-viscoplastic materials. The growth rate of each perturbation mode is obtained as an outcome of the model. This analysis can provide the dominant mode of instability but also gives information concerning the growth of all perturbation modes which lead to the definition of the instability pattern. A probability density function based on the power spectrum of the cross-section profile demonstrates its ability to reproduce the distributions of pre-neck spacing as compared to threshold-based histograms. A definition of pre-neck will be given later. Finally, it is assumed that the surface patterning, obtained at early times from the present analysis, provides the potential sites where necking could be triggered at later stages of deformation.

Previous perturbation analyses have emphasized the emergence of a dominant instability mode from which a characteristic neck spacing was defined. The main outcome of the present analysis is to propose a statistical framework providing information on the distribution of pre-neck spacing. Such information appears to be much richer than those provided by previous works based on a single dominant instability mode.

2. Linear stability analysis model

2.1. Cylindrical bar

A cylindrical bar, of initial length $2L_0$ and radius R_0 is subjected to uniform velocity $V_z = \pm V_0$, applied at the extremities $Z = \pm L_0$, see Fig. 1. Tangential stresses at these extremities are zero. The Lagrangian coordinates of a material point in the round bar are noted (R, θ, Z) in the cylindrical coordinate system associated to

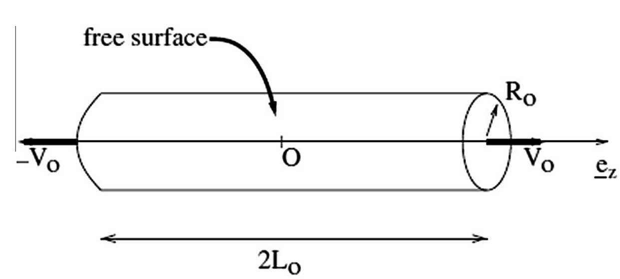

Fig. 1. Schematic representation of the cylindrical bar, of initial length $2L_0$ and of initial radius R_0. Velocities $\pm V_0$ are applied to the extremities $Z = \pm L_0$. The lateral surface $R = R_0$ is traction free.