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Strain gradient effects in periodic flat punch indenting at small scales

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ABSTRACT

Experiments on soft polycrystalline aluminum have yielded evidence that, besides the required punch load, both the size and shape of imprinted features are affected by the scale of the set-up, e.g. substantial details are lost when the characteristic length is on the order of 10 μm. The objective of this work is to clarify the role played by strain gradients on this issue, and to shed light on the underlying mechanisms. For this, indentation by a periodic array of flat punch indenters is considered, and a gradient enhanced material model that allows for a numerical investigation of the fundamentals are employed. During a largely non-homogeneous deformation, the material is forced up in between the indenters so that an array of identical imprinted features is formed once the tool is retreated. It is confirmed that the additional hardening owing to plastic strain gradients severely affects both the size and shape of these imprinted features. In particular, this is tied to a large increase in the mean contact pressure underneath the punch, which gives rise to significant elastic spring-back effects during unloading.

1. Introduction

Flat punch indenting of elastic–plastic solids has earned renewed interest in recent years as a way of rapidly mass producing micron surface features. To achieve high throughput, the surface punching has been evolved into a continuous micro-manufacturing process that relies on imprinting/molding by rolling (referred to as roll-molding, (Lu and Meng, 2013)). However, the underlying mechanisms remains the same. In its simplest form, a flat patterned indentor is pressed into the underlying material and thereby leaving an imprint in the plastically deformed surface once retreated. This classical problem is well-established in the literature, not the least owing to the slip-line field solutions for an rigid perfectly plastic solid by Hill (1950), which has been verified in numerical studies employing conventional plasticity (Nepershin, 2002). Their efforts, along with corresponding studies on pyramidal (Vickers or Knoop), spherical (Brinell) and wedge indentation, have yielded important in-sight into the underlying mechanics, and indentation has become a widely used standard technique in material testing at all scales. It is, however, recognized that indenting at small scales results in increased yield resistance, for materials that deform plastically by dislocation movement, when compared to large scale testing.

When employing indenting (or punching) for manufacturing purposes, the surface imprint is often aimed to represent a counterpart to the indenter as closely as possible. However, a perfect match is complicated by effects such as elastic spring-back, strain gradient hardening, material inertia, and viscosity. Redesign of the punch may improve the imprint, but in general perfectly sharp edges cannot be achieved and some surface curvature must be accepted; this with little noticeable different at large scales. However, deviations from perfectly sharp edges become increasingly evident when the punching process is down-scaled to do micro-manufacturing. Unfortunately, the goal of attaining sharp edges, and abrupt variations in the deformed geometry, are associated with large strain gradients, which lead to the before mentioned increased hardening at micron scale. The explanation for this is now generally accepted to lie in the concept of Geometrically Necessary Dislocations (GND's). When large plastic strain gradients appear GND's must be stored (Ashby, 1970), and this gives rise to free energy associated with the local stress field of the GND's, as-well as increased dissipation when the GND's move in the lattice. At small scales, GND's can become a substantial portion of the total dislocation density which is normally dominated by so-called Statistically Stored Dislocations (SSD's) at larger scale. Thus, a larger amount of energy is required to deform the material at small scales in the presence of gradients, and this leads to an apparent increase in yield stress and strain hardening. To accurately predict the shape and size of imprints made during micro-manufacturing the

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employed material model must therefore represent stresses over the full range of length scales involved.

A vast amount of theoretical literature seeking to encapsulate the experimentally observed gradient effects at micron scale has been put forward, counting both phenomenological models (Aifantis, 1984; Fleck and Hutchinson, 1997, 2001; Gudmundson, 2004; Gurtin and Anand, 2005; Lele and Anand, 2008; Fleck and Willis, 2009a,b), and micro-mechanics based models (Gao et al., 1999; Huang et al., 1999; Gurtin, 2002; Qiu et al., 2003). The higher order theory by Fleck and Willis (2009b) is employed in the current study, and the concept of higher order stresses, work conjugate to the strain gradients, is thus adopted to widen the range of length scales for which the model is valid. The objective is to model an experiment on soft polycrystalline aluminum at small scale, where the impression made by a periodic array of micro-indenters deviates substantially from that observed at larger scales. Through numerical modeling it is the aim to clarify the influence of plastic strain gradients. Moreover, by including unloading the elastic spring-back can be quantified when compared to the surface shape at maximum indentation depth. By choosing a material length parameter of $L_D = 1\ \mu\text{m}$, it is demonstrated that significant gradient effects should be expected for imprinted features with a characteristic length on the order of 10 μm and below. This choice of length parameter are in line with the estimates for the length parameter put forward by Hutchinson (2000) ($L_D \approx 0.25 - 5\ \mu\text{m}$, depending on the gradient type being stretch or rotational), and recently by Danas et al. (2012) ($L_D \approx 0.5 - 1.5\mu\text{m}$).

The paper is structured as follows. The considered boundary value problem is summarized in Section 3, while the material model formulation and numerical procedure are briefly outlined in Sections 2 and 4. A modeling framework capable of predicting the rate-independent material response is employed, and the results are laid out in Section 5. Focus is on shape and size changes to the imprints made onto the plastically deformed surface, as-well as on changes to the loading history due to strain gradient effects. Some concluding remarks are given in Section 6.

2. Strain gradient material models

In spite of indentation being an inherent finite strain problem, a small strain version of the strain gradient plasticity theory by Fleck and Willis (2009b) (tensorial version) is employed in this study as a first approximation. This is considered sufficient for the small indentation depths analyzed. A compact summary of the rate-independent model formulation published by Nielsen and Niordson (2013, 2014) is given below. Throughout, Einstein's summation rule is utilized in the tensor equations and $()_{,i}$ denotes partial differentiation with respect to the spatial coordinate x_i.

2.1. Fundamentals of the Fleck–Willis strain gradient theory

A small strain formulation is employed. The total strain rate is determined from the gradients of the displacement rates; $\dot{\varepsilon}_{ij} = (\dot{u}_{i,j} + \dot{u}_{j,i})/2$, and decomposed into an elastic part, $\dot{\varepsilon}^e_{ij}$, and a plastic part, $\dot{\varepsilon}^p_{ij}$, so that; $\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^p_{ij}$. For a higher order gradient dependent material, involving higher order stresses, the principle of virtual work reads (Gudmundson, 2004)

$$\int_V \left(\sigma_{ij}\delta\varepsilon_{ij} + (q_{ij} - s_{ij})\delta\varepsilon^p_{ij} + \tau_{ijk}\delta\varepsilon^p_{ij,k}\right)\mathrm{d}V = \int_S \left(T_i\delta u_i + t_{ij}\delta\varepsilon^p_{ij}\right)\mathrm{d}S. \tag{1}$$

Here, σ_{ij} is the symmetric Cauchy stress tensor, and $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3$ its deviatoric part. In addition to conventional stresses, the principle of virtual work incorporates the so-called micro-stress tensor, q_{ij} (work-conjugate to the plastic strain, ε^p_{ij}), and the higher order stress tensor, τ_{ijk} (work-conjugate to plastic strain gradients, $\varepsilon^p_{ij,k}$). The right-hand side of Eq. (1) thereby includes both conventional tractions, $T_i = \sigma_{ij}n_j$, and higher order tractions, $t_{ij} = \tau_{ijk}n_k$, with n_k denoting the outward normal to the surface S, which bounds the volume V.

The mechanisms associated with dislocation movement and/or storage of geometrically necessary dislocations (GND's) (Ashby, 1970; Gurtin, 2002; Ohno and Okumara, 2007) have been incorporated into the current higher order theory by assuming the micro-stress, q_{ij}, and higher order stresses, τ_{ijk}, to have a dissipative part only, such that; $q_{ij} = q^D_{ij}$, and $\tau_{ijk} = \tau^D_{ijk}$. Thus, assuming the form of the free energy to be

$$\Psi = \frac{1}{2}(\varepsilon_{ij} - \varepsilon^p_{ij})\mathcal{L}_{ijkl}(\varepsilon_{kl} - \varepsilon^p_{kl}) \tag{2}$$

the conventional stresses are derived as; $\sigma_{ij} = \partial\Psi/\partial\varepsilon^e_{ij} = \mathcal{L}_{ijkl}(\varepsilon_{kl} - \varepsilon^p_{kl})$, where $\mathcal{L}_{ijkl}$ is the isotropic elastic stiffness tensor. In this study, all energetic gradient contributions are omitted. The dissipative stress quantities in the plastic regions read (Gudmundson, 2004; Fleck and Willis, 2009b)

$$q^D_{ij} = \frac{2}{3}\frac{\sigma_C}{\dot{E}^p}\dot{\varepsilon}^p_{ij}, \qquad \text{and} \qquad \tau^D_{ijk} = \frac{\sigma_C}{\dot{E}^p}(L_D)^2\dot{\varepsilon}^p_{ij,k} \tag{3}$$

with σ_C and $\dot{E}^p$ identified as the effective stress and the associated effective plastic strain rate, respectively, given by

$$\sigma_C = \sqrt{\frac{3}{2}q^D_{ij}q^D_{ij} + (L_D)^{-2}\tau^D_{ijk}\tau^D_{ijk}}, \quad \text{and} \quad \dot{E}^p = \sqrt{\frac{2}{3}\dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij} + (L_D)^2\dot{\varepsilon}^p_{ij,k}\dot{\varepsilon}^p_{ij,k}}. \tag{4}$$

Here, $\dot{\varepsilon}^p_{ij,k}$ is the gradient of the plastic strain rates, and L_D is the dissipative length parameter which is included for dimensional consistency. The quantities defined in Eqs. (3) and (4) only exists in the plastic regions (in which $\sigma_C = \sigma_F$), while $q^D_{ij} = q_{ij} = s_{ij}$ in the elastic regions, such that the effective stress reduces to the conventional von Mises stress. An isotropic power hardening material is modeled in the present work, with the current flow stress given by

$$\sigma_F[E^p] = \sigma_y\left(1 + \frac{E^p}{\sigma_y/E}\right)^N \tag{5}$$

Here, E is Young's modulus, N is the strain hardening exponent, and σ_y is the initial yield stress. The material parameters used in the simulations are given in Table 1.

To complete the higher order theory, Fleck and Willis (2009b) put forward two minimum principles that delivers the incremental solution to the displacement rate field, $\dot{u}_i$, and plastic strain rate field, $\dot{\varepsilon}^p_{ij}$.

Assume that the current stress/strain state is known in terms of the displacement, u_i, and plastic strain, ε^p_{ij}, fields. The plastic strain rate field, in the subsequent load increment, is thereby determined as; $\dot{\varepsilon}^p_{ij} = \lambda\dot{\varepsilon}^{p*}_{ij}$, where the plastic trial field, $\dot{\varepsilon}^{p*}_{ij}$, follows from the minimum statement (Minimum Principle I in Fleck and Willis, 2009b)

Table 1
Mechanical properties.

Parameter	Significance	Value
σ_y/E	Uniaxial yield strain	0.001
ν	Poisson's ratio	0.3
N	Strain hardening exponent	0.05–0.2