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Shock wave propagation through a model one dimensional heterogeneous medium

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We study the problem of impact-induced shock wave propagation through a model one-dimensional heterogeneous medium. This medium is made of a model material with spatially varying parameters such that it is heterogeneous to shock waves but homogeneous to elastic waves. Using the jump conditions and maximal dissipation criteria, we obtain the exact solution to the shock propagation problem. We use it to study how the nature of the heterogeneity changes material response, the structure of the shock front and the dissipation.

1. Introduction

Shock wave propagation in solids has been extensively studied for a number of decades, see for example Davison (2008) and the references there. Shocks are described as a moving front across which the stress, strain and particle velocity suffer a discontinuity. They occur when subjected to large deformations at high deformation rates as a result of the nonlinear nature of the equation of state.

While shocks are idealized as a discontinuity, in reality they have a structure where the state of stress varies sharply but smoothly across a narrow region to connect to the limiting states. This structure is commonly attributed to time-dependent inelastic process like viscoelasticity (Band, 1960), viscoplasticity (Swegle and Grady, 1985; Armstrong et al., 2007; Johnson and Barker, 1969), twinning etc. In particular, Swegle and Grady (1985) compiled experimental observations of a wide range of metals and showed that the shock structure follows an universal fourth power law – the peak strain rate in the shock is proportional to the fourth power of the jump in stress across the shock wave. They also proposed a viscoplastic constitutive law consistent with this observation. Recently, Molinari and Ravichandran (2004) revisited this analysis following the constitutive framework of Clifton (1971).

The models of shock structure that are mentioned above are ultimately phenomenological and assume that the material is homogeneous. However, most experiments are conducted on polycrystalline media. One would have significant scattering and dispersion of the elastic and inelastic waves in such a media. Grady (1998) explored the scattering of waves in solids as an alternative explanation to the structured shock waves. In this analysis, elastic modes which were treated using a quasi-harmonic approximation and statistical mechanics were coupled to a nonlinear wave propagation problem. It was shown that this theory produced results in accordance with the single shock data for metals. However, the proposed model was not able to predict more complicated loading like two step shocks. A complete discussion on the fourth power law is presented by Grady (2010).

Structured shocks have also been examined in strongly heterogeneous media, and they do not display the fourth power law in general (Grady, 2010). Zhuang et al. (2003) observed a second power law in periodically layered composites. This work also highlighted the role of scattering by using stress sensors interior to the specimen. Vogler et al. (2012) reinterpreted the observations and suggested an exponent of 2.4 (instead of 2). They also found an exponent of 2–3 in particulate composites and linear relation in granular media. These different exponents in composites are attributed to the scattering of shock waves (as opposed to elastic waves).

The scattering of elastic waves (in linear media) has been widely studied both experimentally and theoretically. Much is known about periodic media where resonances create a highly frequency dependent response through the use of the Bloch–Floquet theory (Sun et al., 1968; Lee and Yang, 1973; Nayfeh, 1995). There is also an understanding of random media and how multiple scattering leads to diffusive response (Ryzhik et al., 1995).

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In contrast, much less is known about nonlinear wave propagation in heterogeneous solids. Chen et al. (2004) adapted the Bloch–Floquet analysis to study plate impact on a periodically layered medium. They developed an analytic solution for the linear case, and used it to obtain an approximate solution for the nonlinear case by making an ansatz about the wave reflections and matching the impedence to nonlinear response. There is also an extensive study of interactions at individual interfaces (Davison, 2008, ch. 9) considering characteristic solutions and Riemann invariants. Since the system also contains rarefaction waves, the interaction happens over a zone and this makes the analysis quite involved.

In this work, we build on the study of individual interactions to understand *the collective response of a heterogeneous medium with multiple interfaces*. Specifically we consider a one-dimensional piecewise homogeneous material with perfectly bonded interfaces. In a typical shock process, the loading happens along the material's Hugoniot while the unloading happens along an isentrope. This leads to a system of shocks and rarefaction waves traveling in the medium. The interaction between shocks, rarefaction waves and interfaces happen over a zone and the solutions are not piecewise uniform. To keep the problem tractable, we idealize the equation of state of each segment in a piecewise affine manner so that there are no rarefaction waves and elastic waves propagate homogeneously, and the heterogeneity is limited to shock waves. We also assume isothermal conditions for simplicity.

It is customary in the study of shock waves to specify an empirical (often linear, (Ruoff, 1967)) relation between the shock speed and particle velocity. We follow Knowles (2002) instead and specify the equation of state as a relation between stress and strain, and supplement it with a kinetic relation that relates the rate of dissipation at the shock front to the shock speed. This framework was introduced in the study of phase transitions (Abeyaratne and Knowles, 1991, 1992), but has also been shown to be useful in the study of shocks (Knowles, 2002; Niemczura and Ravi-Chandar, 2011). In our context, this framework allows us to quickly identify parameters that simplify rarefaction waves and make the elastic waves propagate homogeneously.

We also neglect the structure of the shock, and treat it as a discontinuity. It is known (Abeyaratne and Knowles, 1992) that the kinetic relation can be chosen such that the dissipation at the discontinuity is exactly equal to that of the dissipation in structured shocks. It has also been recently shown (Tan and Bhattacharya, in preparation) that the equivalent sharp discontinuity treatment is appropriate when the length-scale of the heterogeneous media is large compared to the inherent length-scale of the structure shock.

There are a number of powerful numerical methods that can be used in the study of shock waves in one and higher dimensions for detailed empirical material models (see for example Zukas, 2004). These can be used to gain detailed information in specific examples. Our approach using an idealized model is unable to provide such high fidelity information. Instead, the simplified framework that we propose can provide important insight and understanding about a broad range of phenomena. Further, every numerical method has limited resolution (even if it is extremely fine), and this becomes an issue when one has multiple interfaces and reflections. Our results can be used to benchmark these numerical studies.

After recalling the governing equations in Section 3, we study the interaction between a shock wave and an interface in Section 4. Since there are no rarefaction waves, solutions are piecewise uniform in space–time and the interaction leads to a Riemann problem. We are able to solve this Riemann problem analytically. We show that increasing compressibility dissipates the shock while decreasing compressibility intensifies the shock. We extend the analysis to semi-infinite media in Section 5. We show that the shock speed and state of stress in any segment depends only on the properties of the first and that segment.

We turn to the impact of a finite medium in Section 6. We take advantage of the fact that the solution is piecewise constant, and thus it only remains to follow the shock and elastic waves. We propose a new object oriented algorithm to solve this problem exactly. In short, we follow each (elastic and shock) wave and account for all interactions explicitly. We use this method in Section 7 to study the influence of parameters like number of interfaces, arrangement of layers in the target and length of the impactor on the dynamics of the problem on particle velocity profiles, shock structure and effective shock velocities. We use it to provide insights into the optimal arrangement for enhanced attenuation. We conclude in Section 8 with a discussion of the main results.

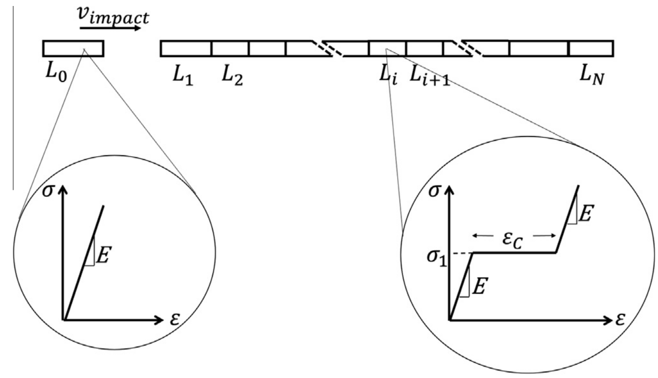

Fig. 1. Schematic representation of the impact problem.

2. Problem statement

We analyze a plate impact induced shock propagation through an idealized nonlinear heterogeneous material. Fig. 1 provides a schematic illustration of the problem. We use the sign convention that compression is positive. We have a linearly elastic impactor traveling at the speed v_{impact} hit a heterogeneous medium or target. The right edge of the ensemble and the left edge of the impactor are free, and the impactor is free to separate from the target.

The heterogeneous medium or target is made of N segments or elements. All the interfaces are perfectly bonded. Each element is made up of a material that follows a piecewise affine stress - strain curve as shown in Fig. 1.[1] Real materials have an effective stress–strain curve that is characterized by an elastic linear region, followed by an yield or Hugoniot elastic limit, and in turn followed by a convex increasing stiffening nonlinear response (Marsh, 1980). We idealize this behavior using a piecewise affine curve. This allows the problem to be simple enough for detailed analysis while retaining the essential features like wave–wave and wave-boundary interactions. Specifically, it collapses rarefaction waves on to unloading shocks. A further idealization is that each material has the same yield strength (σ_1), Young's modulus (E) and density (ρ). So the material is elastically homogeneous. However, each material has a different compressibility ε_c and thus material is heterogeneous with respect to shock waves.

We assume for simplicity that the problem is isothermal.

3. Governing equations

We work in a Lagrangian setting. We denote particle velocity, strain and stress at the particle X in the reference configuration

[1] It is customary in the study of shocks to specify the constitutive relation as a (often linear) relation between the shock speed and particle velocity. We report this relation later for our material.