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Tensegrity deployment using infinitesimal mechanisms

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ABSTRACT

Kinematic properties of tensegrity structures reveal that an ideal way of motion is by using their infinitesimal mechanisms. For example in motions along infinitesimal mechanisms there is no energy loss due to linearly kinetic tendon damping. Consequently, a deployment strategy which exploits these mechanisms and uses the structure's nonlinear equations of motion is developed. Desired paths that are tangent to the directions determined by infinitesimal mechanisms are constructed and robust nonlinear feedback control is used for accurate tracking of these paths. Examples demonstrate the feasibility of this approach and further analysis reveals connections between the power and energy dissipated via damping, infinitesimal mechanisms, speed of the motion, and deployment time.

1. Introduction

Classical tensegrity structures (Fig. 1) are assemblies of flexible elements, called tendons, and disjoint bars (Snelson, 1996). This combination gives tensegrity structures a fascinating form, with the disconnected bars apparently floating in a network of tendons. The *tens*ioned tendons appear to give int*egrity* to the structure, hence the acronym *tensegrity* (Sadao, 1996). Readers interested in this concept's evolution and extensions that include connected bars and other rigid bodies may consult Juan and Tur (2008), Skelton and DeOliveira (2009) or Sultan (2009a). In this article the structures of interest are defined via key properties and modeling assumptions.

The key defining property of tensegrity structures, identified by early tensegrity researchers (Calladine, 1978; Pellegrino and Calladine, 1986), is that they can achieve equilibrium configurations under *zero* external actions (i.e. forces or torques) and with *all* tendons in tension. This property is called prestressability and these equilibrium configurations, prestressable configurations (see Tibert and Pellegrino (2003a) for a review of methods to find such configurations). An immediate consequence of prestressability is that the structure is statically indeterminate at any prestressable configuration, i.e. the equilibrium equations have multiple solutions for the internal forces.

Another key property of classical tensegrities is that they have kinematically indeterminate prestressable configurations with internal infinitesimal mechanisms. A configuration is kinematically indeterminate if infinitesimal displacements are possible with no changes in the lengths of the structural members (Calladine, 1978). Such displacements are called infinitesimal mechanisms. The adjective "internal" is sometimes used to emphasize the fact that tensegrity infinitesimal mechanisms are intrinsic to the structure and not due to effects such as rigid body motions, which involve large displacements with no changes in the lengths of the structure's members (see Pellegrino and Calladine (1986) for details on this topic). Note that, in general, i.e. not limiting the discussion to tensegrity, mechanisms lead to changes in the structural member lengths that are at least of second order in terms of displacements and are classified according to this order, culminating with finite mechanisms, which result in zero changes in the structural member lengths for large displacements, thus being similar to rigid body motions in this respect (the interested reader may refer to Pellegrino and Calladine (1986), Calladine and Pellegrino (1991) or Vassart et al. (2000) and references therein).

The existence of mechanisms is a major advantage for structures which require change of configuration (e.g., morphing structures, robots, deployable structures, etc.). Indeed, mechanisms enable configuration changes without modifications in the internal member lengths. For infinitesimal mechanisms this is of course valid for infinitesimal displacements while for finite mechanisms it is valid even for large displacements. A structure with mechanisms has increased "mobility" compared to structures without mechanisms, making it more amenable to dynamic applications which involve configuration changes. Clearly, this is true for *any* structure with mechanisms, including articulated assemblies composed only of bars. In structures with tendons and mechanisms, the mechanisms provide another advantage for dynamic applications. Specifically for tensegrity, the energy dissipated via linearly

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Fig. 1. A tensegrity sculpture by Kenneth Snelson.

kinetic tendon damping is zero in motions that occur along infinitesimal mechanisms. It can also be shown, using a simple approximation, that the variation in the potential elastic energy of a tensegrity structure when displacements along infinitesimal mechanisms occur is small. More generally, i.e. not limiting the discussion to tensegrity and infinitesimal displacements, if a structure with tendons has finite mechanisms the energy dissipated via linearly kinetic tendon damping and the potential elastic energy variation are both zero for large displacements along these mechanisms because tendon lengths do not change (of course the above rationale assumes that tendon rest-lengths are constant).

The previous paragraph outlined major advantages mechanisms provide for dynamic applications, especially for structures with tendons like tensegrity, emphasizing that motions along mechanisms are desired. However, kinematic analysis, which is used to identify mechanisms, is basically a geometric study and by itself cannot address the questions if such motions are feasible and how they can be achieved. For this purpose, the dynamic equations of motion must be employed. Furthermore, for large displacement applications such as deployment, nonlinear dynamics equations are required.

Nonlinear ordinary differential equations were used to model tensegrity's dynamics in a deployment strategy in Sultan and Skelton (2003). In that work mechanisms were not exploited. Instead, the system was controlled using tendons to maintain the state space trajectory of the deployment process close to an equilibrium manifold. The evolution of the structure was quasi-static, facilitating satisfaction of structural integrity and collision avoidance constraints. Sultan et al. (2002) also developed a non quasi-static reconfiguration procedure which exploits the mathematical structure of the nonlinear equations of motion and symmetrical tensegrity configurations. Tendon control and, in some cases, external torque control applied to a rigid element of the structure, was used to achieve symmetrical motions. Working on other tensegrity deployment problems, Tibert and Pellegrino (2002, 2003b) disputed tendon control claiming that it is technologically complicated and proposed deployment using foldable/telescopic struts. A disadvantage of this strategy is that the structure has slack tendons until fully deployed. Fest et al. (2004) studied the potential of telescopic struts in the shape control of a tensegrity structure assuming quasi-static evolution. Motro et al. (2006) proposed deployable tensegrity rings that can be assembled in pedestrian bridges. Smaili and Motro (2007) investigated folding of tensegrity systems by creating finite mechanisms. Finite mechanisms have also been exploited in Rhode-Barbarigos et al. (2012) in a study of ring modules (see also Rhode-Barbarigos et al. (2010)) for a deployable footbridge, where the structure is deployed assuming sufficient damping and quasi-static evolution. A key idea in using finite mechanisms in tensegrity deployment is to "activate" these mechanisms, for example by changing the lengths of telescopic struts or tendons. The main disadvantage associated with this procedure is that instabilities are introduced when finite mechanisms are created. These issues are amply described in Motro (2003) Chapter 6.

As emphasized in the above, many successful deployment methods are quasi-static. The structure's generalized velocities and accelerations are very small and the state space trajectory of the deployment process is maintained close to an equilibrium set. Quasi-static strategies are very effective when damping is large because it naturally facilitates small accelerations and velocities. This explains the success of quasi-static deployment procedures in the presence of considerable damping. However, for many applications one would actually like to *reduce* damping because of its detrimental effects. Damping is a thermodynamically irreversible process which may result in large energy dissipation and non-desirable thermal effects. On one hand, it is well known that these effects are particularly damaging for tendons composed of certain materials such as elastomers. On another hand, such materials may actually be required, especially in deployment applications. This is so because deployment requires large geometry changes that may easily translate into the requirement that tendons tolerate large strain variations, as it will be revealed by examples included in this article. The requirement for large strains is fulfilled by tendons made of elastomers. Therefore, developing deployment strategies in which the energy dissipated via tendon damping is small is important. Also, quasi-static deployment strategies are inherently slow because they require small velocities and accelerations that usually result in long deployment times. This can be reduced by solving a constrained optimization problem aimed at minimizing the deployment time, which is not an easy task (see Sultan and Skelton (2003) for such an example).

This article directly addresses the last two issues. A fast deployment procedure, specifically focused on achieving small energy dissipation via tendon damping, is developed. Because in motions along infinitesimal mechanisms the energy dissipated via linearly kinetic tendon damping is zero, a natural solution is to use these mechanisms for deployment. For this purpose, desired paths that are tangent to the directions determined by infinitesimal mechanisms are created. The requirement of quasi-static motion is eliminated and the desired paths are not constrained to be close to an equilibrium set. The amplitude of the structure's motion is also not restricted to small variations around equilibria, therefore nonlinear ordinary differential equations are used to describe the structure's dynamics. Furthermore, robust nonlinear feedback controllers are designed to guarantee that the state space trajectories of the deployment process, called actual paths, track the desired paths in the presence of uncertainties. These controllers use only torques and eventually forces applied to the bars, which are technologically easy to implement. Because the actual paths follow closely trajectories that are tangent to infinitesimal mechanisms it is expected that the energy dissipated via tendon damping is small. Examples reveal the feasibility of the procedure on a tensegrity simplex as well as on a much more complex tensegrity tower. Correlations between the power dissipated via damping, the speed of the motion, and the infinitesimal mechanisms, as well as the influence of the deployment time on the energy dissipated via damping are analyzed. Issues related to material selection, structural integrity, robustness of the design are also amply discussed.

2. Mathematical modeling, prestressability, mechanisms

2.1. Modeling assumptions

The bars are stiff in comparison with the tendons and the mass of each bar is large relative to the mass of each tendon. Therefore,