International Journal of Solids and Structures 51 (2014) 3669–3678

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Variational analysis for angle-ply laminates with matrix cracks

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ARTICLE INFO

Article history:
Received 10 November 2013
Received in revised form 14 April 2014
Available online 17 July 2014

Keywords:
Transverse cracking
Angle-ply laminate
Stiffness reduction
Fiber orientation

ABSTRACT

A stress-based variational model is developed to study stiffness reduction and stress distribution in angle-ply laminates $[\theta_{2l}/\theta_{1m}/90_n]_s$ with matrix cracks. The inter-laminar shear stresses between 90° and θ_1°-plies and between $\theta^\circ{}_1$ and $\theta^\circ{}_2$-plies, respectively, are assumed to be in the form of general functions. The normal stresses σ_x in $\theta^\circ{}_1$-plies and $\theta^\circ{}_2$-plies are introduced with partition coefficient λ for solving the problem of statically indeterminate boundary because the normal stresses σ_x cannot be obtained by using the condition of statics due to the loads at the boundary for each uncracked layer. This leads to expressions derived from equilibrium equations and boundary conditions for stress components in terms of the general functions and the partition coefficient. The governing equations for the general functions and the partition coefficient are derived by using a variational approach with the principle of minimum complementary energy. As an application, reduction of Young's modulus for different laminates is evaluated and compared with available experimental results. Distributions of in-planar and inter-laminar stresses are also presented by means of the finite difference method. The results show that the present approach is suitable to analyze stiffness reduction for multi-angle-ply laminates with transverse cracks in 90° layer.

1. Introduction

General laminates are made of unidirectional fiber layers oriented at different angles. It is load or cyclic accumulation can induce matrix cracking in some layers. Early experimental data for laminated composites with lay-up $[0_m/90_n]_s$ have shown that matrix cracks in 90° layer reduce stiffness and change Poisson's ratio. Many researchers have presented a lot of approaches to study the properties of the damage due to the matrix cracking in the laminates, such as Shear-lag model (Reifsnider et al., 1983; Highsmith and Reifsnider, 1982; Katerelos et al., 2005; Amara et al., 2005; Berthelot et al., 1996; Bouiazza et al., 2007; Tounsi et al., 2005; Berthelot and Le Corre, 1999) and variational analysis (Hashin, 1987; Rebière et al., 2001; Vinogradov and Hashin, 2005). As the early shear-lag model was generally prone to large errors when comparing with experimental data, several modified models have been proposed based on the assumption of displacement in the 90° plies by Katerelos et al. (2005), Amara et al. (2005), Berthelot et al. (1996), Bouiazza et al. (2007) and Tounsi et al. (2005). Using the progressive shear models and the parabolic analysis model, Berthelot and Le Corre (1999) showed that the results for progressive shear model are in good agreement with simulated results obtained by the finite element method. The research of Tounsi and Amara (2005) shows that the modified shear-lag model can further be used to analyze stiffness degradation in aged cross-ply laminate with transverse cracks under hygrothermal condition. For more accurate stress analysis, a variational approach based on the principle of minimum complementary or potential energy is developed by Hashin (1987) who obtained the minimum average threshold of stiffness degradation for laminates $[0_m/90_n]_s$. Highsmith and Reifsnider (1982) studied the stiffness-reduction mechanisms in composite laminates, but there are large errors between Poisson's ratio in theoretical results and experimental observations in their work In addition, Rebière et al. (2001) showed that there are large errors in the inter-laminar stresses calculated with some models and approaches, such as variational approach used by Hashin (1987), Perturbation stress functions of SUPPRIMER with a parabolic variation (model 1) and a second order polynomial (model 2) and finite-element method. The variational analysis was utilized by Vinogradov and Hashin (2005) to estimate the stress fields in a cracked laminate subjected to an applied load and a temperature change. Their work showed that the probability density function for the specific surface energy requires modification when applying to different laminates. Huang et al. (2011) obtained an exact solution for stresses in cracked laminates of $[\theta_m/90_n]_s$ with

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http://dx.doi.org/10.1016/j.ijsolstr.2014.06.028
0020-7683/

consideration of the series expansion form of sinusoidal functions for inter-laminar shear stresses in 90° and 0° plies. Meanwhile, the results of interlaminar stress show that delamination occurs near the splitting location.

Mechanical properties of $[\theta_m/90_n]_s$ laminates with transverse cracks have been reported in many articles. Zhang et al. (1992) and Kashtalyan and Soutis (2002) discussed stiffness degradation in angle-ply laminates with matrix cracks based on a 2-dimensional shear-lag model. Nairn and Hu (1992) used a new 2-dimensional stress analysis to calculate total strain energy, effective modulus and energy release rate for $[(S)/90_n]_s$ laminates having matrix cracks and delamination, and Farrokhabadi et al. (2011) used a generalized micromechanical approach to analyze stress distributions and energy release rate of laminates with matrix cracking. Using shear-lag model and variational model, Joffe and Varna (1999), Joffe et al. (2001) and Amara et al. (2006) analyzed stiffness reduction in laminates caused by cracks in 90° layers. Also, Pradhan et al. (1999) used a 3-D finite-element method to evaluate the degradation of stiffness of $[\theta_m/90_n]_s$ laminates with transverse cracking in 90° ply. Further, Zhang and Minnetyan (2006) examined theoretically the degradation of stiffness and the energy release rate by using a displacement-based variational approach based on the hypothesis of displacement function of sub-laminates for $[\theta_m/90_n]_s$ laminates with transverse cracking and local delamination. In addition, Vinogradov and Hashin (2010) used the principle of minimum complementary energy to analyze stiffness reduction of angle-ply laminates with matrix cracks in middle laminate.

The above research models and approaches are mainly used to analyze $[\theta_m/90_n]_s$ laminates with matrix defects, but the effects of shear stress acting in the plane of each ply are omitted when using the average properties of $\pm\theta$ plies to analyze the $[\pm\theta/90_n]_s$ laminates. Tong et al. (1997) show that in-planar stresses for uncracked laminas of +45° ply and −45° ply are different in their finite element analysis. Since there is the shear stress in each ply, this makes it difficult to analyze in greater depth. Zhang and Herrmann (1999) proposes a theoretical model based on effective in-plane stresses and strains as well as equivalent constraint model (ECM) for the prediction of the elastic properties of a general symmetric laminate containing multilayer matrix cracks, but it is difficult to analyze the inter-laminar stresses for the laminates with matrix cracks. Li and Hafeez (2009) uses the principle of minimum complementary energy to analyze the stress of multi-angle ply laminates with matrix cracks without considering the effects of shear stress acting in the plane of each ply. In this work, inter-laminar shear stresses are assumed to be in the form of general functions of $[\theta_{2l}/\theta_{1m}m/90_n]_s$ laminates, which leads to general expressions derived from equilibrium equations and boundary conditions for stress components in the laminate. Based on the principle of minimum complementary energy, the governing equations subjected to the condition of statically indeterminate boundary are derived for the general functions and partition coefficient. By means of the finite difference method, reduction of Young's modulus for various glass/epoxy laminates are evaluated and compared with available experimental results. Distributions of in-planar and inter-laminar stresses are also presented in this work. The present approach is suitable for analysis of damage evolution about multi-angle ply laminates with transverse cracks in 90° layer.

2. Fundamental equations, boundary conditions and the form of solution

Early experimental observations (Highsmith and Reifsnider, 1982) showed that as crack density increases, the transverse cracks in 90° layers are evenly spaced along the length of the laminates, and the cracks extend across the entire width and occupy the whole thickness of 90° layers. A characteristic cracked element in composite laminates with lay-up $[\theta_{2l}/\theta_{1m}m/90_n]_s$ under prescribed tension $\overline{\sigma_x}$ is shown in Figs. 1 and 2. By virtue of symmetry, the half ($z \geqslant 0$) of the cracked element is analyzed. In the cracked element, $h = t_1 + t_2 + t_3$, where $t_i = n_i t_0$ for $i = 1, 2, 3$ and $n_1 = n$, $n_2 = m$, $n_3 = l$, and t_0 is the thickness of a single layer. The superscripts (1), (2) and (3) represent the 90°, $\theta^\circ{}_1$ and $\theta^\circ{}_2$ layers, respectively. For the statically indeterminate problem, layers (2) and (3) are subjected to tension $\frac{h}{t_2}\overline{\sigma_x}\lambda$ and $\frac{h}{t_3}(1-\lambda)\overline{\sigma_x}$, respectively, at the boundary. λ is unknown partition coefficient. The normal stresses of (2) and (3) layer at the boundary have

$$t_2\sigma_x^{(2)}\big|_{x=L_1} + t_3\sigma_x^{(3)}\big|_{x=L_1} = h\overline{\sigma_x}$$

Without body forces, equilibrium equations are given as

$$\begin{cases} \frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} = 0 \\ \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} = 0 \\ \frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} = 0 \end{cases} \tag{2.1}$$

and boundary conditions are expressed by

$$\begin{aligned} &\sigma_x^{(1)}\big|_{x=\pm L_1} = 0, \quad \tau_{xz}^{(1)}\big|_{z=0} = \tau_{yz}^{(1)}\Big|_{z=0} = 0 \\ &\sigma_z^{(3)}\big|_{z=h} = 0, \quad \tau_{xz}^{(3)}\big|_{z=h} = \tau_{yz}^{(3)}\Big|_{z=h} = 0 \\ &\tau_{xy}^{(i)}\Big|_{x=\pm L_1} = 0, \quad \tau_{xz}^{(i)}\big|_{x=\pm L_1} = 0, \quad (i = 1,2,3) \\ &t_2\sigma_x^{(2)}\big|_{x=\pm L_1} + t_3\sigma_x^{(3)}\big|_{x=\pm L_1} = h\overline{\sigma_x} \end{aligned} \tag{2.2}$$

When the laminate is subjected to uniaxial tension in x direction, the resultants per unit length should be $N_y = 0$ and $N_{xy} = 0$, which can be reduced to

$$\int_0^{t_1}\sigma_y^{(1)}dz + \int_{t_1}^{t_1+t_2}\sigma_y^{(2)}dz + \int_{t_1+t_2}^{h}\sigma_y^{(3)}dz = 0 \tag{2.3}$$

$$\int_0^{t_1}\tau_{yx}^{(1)}dz + \int_{t_1}^{t_1+t_2}\tau_{yx}^{(2)}dz + \int_{t_1+t_2}^{h}\tau_{yx}^{(3)}dz = 0 \tag{2.4}$$

Traction continuity at the interfaces requires

$$\begin{aligned} &\tau_{zx}^{(1)}\big|_{z=t_1} = \tau_{zx}^{(2)}\big|_{z=t_1}, \quad \tau_{zx}^{(2)}\big|_{z=t_1+t_2} = \tau_{zx}^{(3)}\big|_{z=t_1+t_2} \\ &\tau_{zy}^{(1)}\Big|_{z=t_1} = \tau_{zy}^{(2)}\Big|_{z=t_1}, \quad \tau_{zy}^{(2)}\Big|_{z=t_1+t_2} = \tau_{zy}^{(3)}\Big|_{z=t_1+t_2} \\ &\sigma_z^{(1)}\big|_{z=t_1} = \sigma_z^{(2)}\big|_{z=t_1}, \quad \sigma_z^{(2)}\big|_{z=t_1+t_2} = \sigma_z^{(3)}\big|_{z=t_1+t_2} \end{aligned} \tag{2.5}$$

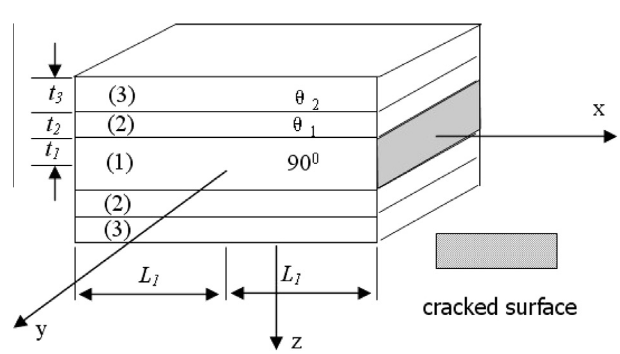

Fig. 1. Geometry of cracked element.