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Love wave dispersion in pre-stressed homogeneous medium over a porous half-space with irregular boundary surfaces

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ABSTRACT

The paper investigates the existence of Love wave propagation in an initially stressed homogeneous layer over a porous half-space with irregular boundary surfaces. The method of separation of variables has been adopted to get an analytical solution for the dispersion equation and thus dispersion equations have been obtained in several particular cases. Propagation of Love wave is influenced by initial stress parameters, corrugation parameter and porosity of half-space. Velocity of Love waves have been plotted in several figures to study the effect of various parameters and found that the velocity of wave decreases with increases of non-dimensional wave number. It has been observed that the phase velocity decreases with increase of initial stress parameters and porosity of half-space.

1. Introduction

The nature of different seismic waves is studied in theoretical seismology and also has some practical importance in the field of Civil Engineering, Rock Mechanics and Geophysical Prospecting. The propagation of Love waves in a homogeneous medium over semi-infinite porous medium has importance in earthquakes engineering and seismology on account of the occurrence of porosity, inhomogeneity in the crust of the Earth as the Earth is supposed to be made up of different layers. In the beneath of Earths surface the porous layer is naturally found. In general the pores contain hydrocarbon deposition such as gas and oil. Most oil and gas deposits are found in sandstone or limestone is very much like a hard sponge, full of holes but not compressible. These holes or pores can contain water or oil or gas and rock will be saturated with one of these three. The holes are much tinier than sponge holes but they are still holes and they are called porosity and the layer is called porous layer. The studies of Love wave propagation in a liquid saturated porous medium play an important role in the field of Geophysical problems leading to the exploration of oil and underground water. The propagation of Love wave in elastic media with irregular boundary surfaces is also important leading to better understanding and prediction of seismic wave behaviour at continental margins, mountain roots, etc. Propagation of surface waves in a homogeneous medium over an inhomogeneous elastic half-space are well known and prominent feature of wave theory. Quite a large amount of information about propagation of seismic waves is documented in well-known books written by Biot (1965), Ewing et al. (1957), Gubbins (1990), etc.

The stress generated in a medium, referred as initial stress and may be developed in media due to natural phenomena or by any artificial stress. The Earth may be assumed as an elastic solid layered medium under high initial stresses. These initial stresses contribute a significant influence on elastic waves produced by earthquakes. Many researchers have devoted their work to solve various problems on the propagation of surface waves. Dey et al. (1996) discussed about propagation of Love waves in heterogeneous crust over a heterogeneous mantle. They (Dey et al., 2004) also studied about propagation of Love waves in an elastic layer with void pores. The influence of anisotropy on the Love waves in a self-reinforced medium was formulated by Pradhan et al. (2003). Sharma (2004) established a mathematical expression about wave propagation in a general anisotropic poroelastic medium with anisotropic permeability phase velocity and attenuation. Kalyani et al. (2008) have made finite difference modeling of seismic wave propagation in monoclinic media.

Propagation of seismic waves in layered media bounded by different forms of irregular boundaries has been investigated by many authors. Wolf (1970) observed the propagation of Love waves in layers with irregular boundaries. The dispersion equation for Love wave due to irregularity in the thickness of non-homogeneous crustal layer was obtained by Chattopadhyay

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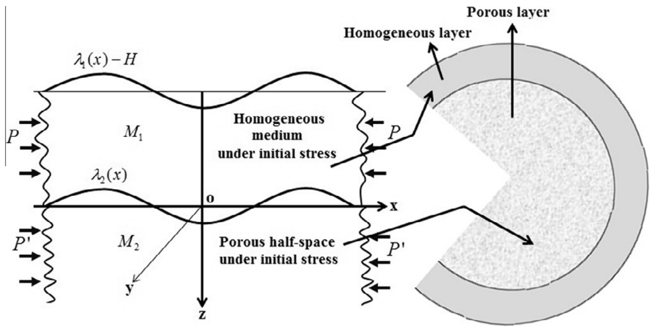

Fig. 1. Geometry of the problem.

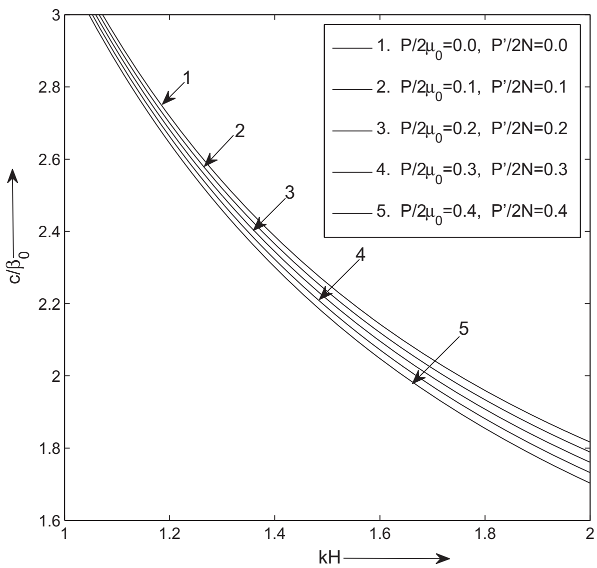

Fig. 2. Case I: dimensionless phase velocity $\frac{c}{\beta_0}$ as a function of dimensionless wave number kH of Love waves for different values of initial stresses $\frac{P}{2\mu_0}$ and $\frac{P'}{2N}$.

(1975). The influence of irregularity and rigidity on the propagation of torsional wave was developed by Gupta et al. (2010a). Chattaraj et al. (2013) introduced the dispersion equation of Love wave, propagating in an irregular anisotropic porous stratum under initial stress. Chattopadhyay et al. (2010) studied about propagation of SH waves in an irregular non-homogeneous monoclinic crustal layer over a semi-infinite monoclinic medium. Propagation of Love wave at a layered medium bounded by irregular boundary surfaces was developed by Singh (2011). Dispersion of horizontally polarized shear waves in an irregular non-homogeneous self-reinforced crustal layer over a semi-infinite self-reinforced medium was derived by Chattopadhyay et al. (2013). Liu and He (2010) studied the properties of Love waves in layered piezoelectric structures.

A good amount of research has been done by many authors in the field of Love wave propagation. Ke et al. (2006) studied the Love waves in an inhomogeneous fluid saturated porous layered half-space with linearly varying properties. Ghorai et al. (2010) shown the Love waves in a fluid-saturated porous layer under a rigid boundary and lying over an elastic half-space under gravity. Propagation of Love waves in an orthotropic Granular layer under initial stress overlying a semi-infinite Granular medium was established by Ahmed and Abd-Dahab (2010). Gupta et al. (2010b) discussed the effect of initial stress on propagation of Love waves in an anisotropic porous layer. Disturbance of SH-type waves due to discontinuity of shearing stress in a visco-elastic layered half-space was formulated by Pal and Sen (2011). Kielczynski et al. (2012) observed the effect of a viscous liquid loading on Love wave propagation.

Recently numerous papers have been done by many researchers in the field of wave propagation. Such as Gupta et al. (2013a) discussed about the propagation of Love waves in a non-homogeneous substratum over an initially stressed heterogeneous half-space. Love waves in the fiber-reinforced layer over a gravitating porous half-space was investigated by Chattaraj and Samal (2013). Possibility of Love wave propagation in a porous layer under the effect of linearly varying directional rigidities was introduced by Gupta et al. (2013b). SH-type waves dispersion in an isotropic medium sandwiched between an initially stressed orthotropic and heterogeneous semi-infinite media were studied by Kundu et al. (2013). Manna et al. (2013) formulated Love wave propagation in a piezoelectric layer overlying in an inhomogeneous elastic half-space. Bacigalupo and Gambarotta (2014) discussed about second-gradient homogenized model for wave propagation in heterogeneous periodic media. Propagation of Love wave in fiber-reinforced medium lying over an initially stressed orthotropic half-space was obtained by Kundu et al. (2014).

In this paper, the propagation of Love wave in a homogeneous irregular layer over an elastic porous half-space has been briefly studied. Both the layer and half-space are considered under the effect of initial stress. The dispersion relations have been derived in some particular cases by taking irregular boundary surfaces, $a_1 cos(bx)$ and $a_2 cos(bx)$, where a_1 and a_2 are amplitudes of the surfaces. The influences of porosity, initial stress parameters and corrugation parameter have discussed graphically.

2. Mathematical formulation of the problem

We consider an initially stressed (P) elastic homogeneous layer $M_1 : \lambda_1(x),\ \lambda_1(x) - H \leqslant z$ over an initially stressed (P') porous half-space, $M_2 : \lambda_2(x) \leqslant z \leqslant \infty$ as shown in Fig. 1, where H can be assumed as the average thickness of the upper layer. The x axis and y axis are considered as two perpendicular Cartesian coordinates lying horizontally and vertically coordinate with positive direction pointing downward can be taken as z axis. Where $\lambda_1(x)$ and $\lambda_2(x)$ are continuous functions of x independent of y and consider as the irregular boundaries of the layer. The x axis is parallel to the direction of propagation of waves. So, the non-zero field of quantities representing the motion are only function of $x,\ z$ and time t.

The functions, $\lambda_j(x)$ can be taken as periodic in nature and their Fourier series expansions are provided as Singh (2011)

$$\lambda_j(x) = \sum_{n=1}^{\infty}\left(\lambda_n^j e^{inpx} + \lambda_{-n}^j e^{-inpx}\right), \quad j = 1, 2. \tag{1}$$

Here the Fourier series expansion coefficients are λ_n^j and λ_{-n}^j, the wave number is p, $\frac{2\pi}{p}$ is the wavelength, n is the order of series expansion and $i = \sqrt{-1}$. We assumed a very small amplitude of irregular boundaries compared with the wavelength.

3. Dynamics of upper homogeneous layer

The upper layer of the formulated problem is considered as initially stressed homogeneous medium. Let $u_1,\ v_1$ and w_1 be the displacements along $x,\ y$ and z directions respectively. First, we look for the equations governing the propagation of Love wave in homogeneous elastic medium. In this medium waves are propagating along x axis. The equations of motion for a homogeneous elastic solid in the absence of body forces in component form are (Biot, 1965)