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On the relationship of the shear deformable Generalized Beam Theory with classical and non-classical theories

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ABSTRACT

The possibility to establish clear relationships between the results of the Generalized Beam Theory (GBT) and those of the classical beam theories is a crucial issue for a correct theoretical positioning of the GBT within the other existing beam theories as well as for the application of the GBT in the current engineering practice. With this in mind, the recovery of classical and non-classical beam theories within the framework of the GBT is presented in this paper. To this purpose, a new formulation of the GBT with shear deformation is conceived. Particularly, the formulation recently proposed by the authors is here modified by introducing new definitions of the kinematic parameters and of the generalized deformations, and extended to the dynamic case. Firstly, it is shown that a suitable choice of the flexural deformation modes allows recovering the Vlasov beam theory, both with and without shear deformation. Also, the analytical solution of the non-uniform torsion problem with shear deformation is given. Then, the recovery of the Capurso beam theory using the nonlinear warping deformation modes is illustrated.

1. Introduction

Thin-walled beams are used in a broad variety of structures, ranging from the aeronautical to the civil engineering. Accordingly, much research has been devoted to the development of effective analysis tools, that combine easy usage and good predictive capabilities, to evaluate their structural behavior. The first important contribution for the analysis of thin-walled beams was the well-known theory developed by Vlasov (1961). Later, Capurso (1964a,b, 1984) generalized the Vlasov theory by introducing the shear deformability along the wall midline. In particular, this was achieved by enriching the warping description, while keeping null in-plane deformation of the cross-section, as in the Vlasov beam. Then, the concept of generalized warping functions has been used further by many authors (see, for example, Bauchau, 1985; De Lorenzis and La Tegola, 2005; Genoese et al., 2014; Ferradi and Cespedes, 2014). On the other hand, in the 80s, Schardt (1989, 1994) proposed the Generalized Beam Theory (GBT), which has been proven to consistently account for cross-section distortion along with the classical kinematics of axial displacement, bending and torsional rotation in a comprehensive fashion. The fundamental idea of the GBT is to assume the displacement field of the beam as a linear combination of predefined cross-section deformation modes multiplied by unknown functions dependent on the beam axial coordinate, called kinematic parameters or generalized displacements. From its original form, many authors have contributed to the improvement of the GBT by extending it beyond its original formulation for open unbranched sections (Dinis et al., 2006; Silvestre, 2007, 2008; Goncalves et al., 2009), by adding geometric nonlinear effects (Davies et al., 1994; Silvestre and Camotim, 2003a; Camotim et al., 2010; Silva and Silvestre, 2007; Silva et al., 2010), by developing beam elements based on semi-analytical solutions (Andreassen and Jonsson, 2013), or by presenting new formulations for the dynamic analysis of open-section members subjected to initial perturbations or acting loads (Bebiano et al., 2013). Moreover, an interesting application of the GBT to analyze cold-formed roof systems has been presented by Braham et al. (2008), an effective equlibrium-based procedure for the reconstruction of the three-dimensional stresses in GBT members by de Miranda et al. (2014), and the discussion of analogies between the GBT and the constrained Finite Strip Method by Adany et al. (2009) and Silvestre et al. (2011).

Recently, a formulation of the GBT for the elastic–plastic analyses of thin-walled members experiencing arbitrary deformations and made of non-linear materials has been developed (Abambres et al., 2013, 2014a) and used for the modal decomposition of equilibrium/collapse configurations in the context of an inelastic member

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analysis (Abambres et al., 2014b). Moreover, a GBT-based method capable of identifying the modal participation of the fundamental deformation modes from a general buckling mode determined by using the Finite Element Method has been presented by Nedelcu and Cucu (2014).

The selection of the cross-section deformation modes (usually referred to as cross-section analysis) has received extensive attention in the research community over the years. On this regard, in the spirit of the semi-variational method, an interesting approach that reverses the classical methodology of GBT cross-section analysis has been proposed by Ranzi and Luongo (2011): firstly an in-plane analysis is carried out by solving a dynamic eigenvalue problem relevant to an inextensible planar frame having the shape of the cross-section middle line, then the warping is evaluated by enforcing the Vlasov unshearability condition. Recently, an extension of this dynamic approach to include also non-conventional (extension and shear) modes has been presented by Piccardo et al. (2013) and a variant, based on a new quadratic functional, by Piccardo et al. (2014).

In the GBT literature, much attention has also been devoted to the shear deformability (Silvestre and Camotim, 2003b, 2004, 2013; de Miranda et al., 2013). In particular, a new formulation of the GBT that coherently accounts for the shear deformation has been recently presented by de Miranda et al. (2013). Guaranteeing a coherent matching between bending and shear strain components of the beam, the new formulation allows to clearly identify the classical degrees of freedom of the beam, an important issue to develop geometrically nonlinear formulations based on corotational approaches (Zagari et al., 2013; Garcea et al., 2009, 2012).

Indeed, notwithstanding the great amount of literature on GBT, in the author's opinion there is still a need for a proper theoretical positioning of the GBT within the framework of the other existing beam theories. This would allow to establish clear relationships between the GBT results and those of the classical beam theories, a crucial issue to apply the GBT in the current engineering practice. An interesting early attempt in this direction, limited to the unshearable Vlasov theory, was presented by Silvestre and Camotim (2002). With this in mind, the recovery of classical and non-classical beam theories within the framework of the GBT is presented in this paper. The starting point is the shear deformable GBT presented by de Miranda et al. (2013), here properly reformulated by introducing different definitions of the kinematic parameters and of the generalized deformations, and extended to the dynamic case. In particular, firstly it is shown how it is possible to reduce the new GBT formulation to the standard shear undeformable GBT. Then, it is shown that a suitable choice of the deformation modes allows to recover the Vlasov beam theory, both with and without shear deformation. On this regard, the analytical solution of the non-uniform torsion problem with shear deformation is given and an example discussing the influence of the shear deformability is presented. Finally, the recovery of the Capurso beam theory using the nonlinear warping deformation modes is illustrated.

The paper is organized as follows. The kinematics of the new GBT is presented in Section 2 and the complete formulation of the GBT for the flexural deformation modes in Section 3. The reduction of the present shear deformable GBT to the classical shear undeformable one is presented in Section 4. Section 5 is devoted to the recovery of the Vlasov beam theory. The GBT formulation for nonlinear warping modes is presented in Section 6 and the recovery of the Capurso beam theory in Section 7. Some final considerations end the paper.

2. Kinematics

The GBT can be viewed as a one-dimensional theory deduced from a parent three-dimensional theory basing on some kinematical ansatzs. In particular, the displacement field of the beam is assumed as a linear combination of predefined cross-section deformation modes multiplied by generalized displacements that depend on the beam axial coordinate. Thus, at the generic time t, the following displacement field is assumed for the generic ith wall of the cross-section (see Fig. 1):

$$d_n(s,z,t) = \boldsymbol{\psi}(s)\mathbf{v}(z,t), \tag{1}$$

$$d_s(n,s,z,t) = \boldsymbol{\xi}(s,n)\mathbf{v}(z,t), \tag{2}$$

$$d_z(n,s,z,t) = \boldsymbol{\omega}(s,n)\mathbf{w}(z,t), \tag{3}$$

where d_n is the displacement orthogonal to the wall midline, d_s is the displacement tangent to the wall midline, d_z is the displacement in the beam axial direction, $\boldsymbol{\psi}$, $\boldsymbol{\xi}$ and $\boldsymbol{\omega}$ are row matrices collecting the assumed cross-section deformation modes (depending only on s and n), and $\mathbf{v}$ and $\mathbf{w}$ are vectors that collect the unknown kinematic parameters (depending only on z and t). In accordance with the hypothesis that the generic wall behaves as a Kirchhoff plate, cross-section deformation modes $\boldsymbol{\xi}$ and $\boldsymbol{\omega}$ are assumed to depend linearly on n in the form:

$$\boldsymbol{\xi}(n,s) = \boldsymbol{\mu}(s) - n\mathring{\boldsymbol{\psi}}(s), \quad \boldsymbol{\omega}(n,s) = \boldsymbol{\varphi}(s) - n\boldsymbol{\psi}(s), \tag{4}$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\varphi}$ are predefined shape functions. Hereinafter, $\mathring{(\,)}$, $(\,)'$ and $\overset{\times}{(\,)}$ denote the derivatives with respect to the s, z and n coordinates, respectively. It can be easily verified that, by a suitable redefinition of the generalized displacements $\mathbf{w}$, the above kinematics coincides with that proposed by de Miranda et al. (2013).

Eqs. (1)–(3) can be recast in the following matrix form:

$$\mathbf{d}(n,s,z,t) = \mathbf{U}(s,n)\mathbf{u}(z,t), \tag{5}$$

where:

$$\mathbf{d} = \begin{bmatrix} d_n \\ d_s \\ d_z \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} \boldsymbol{\psi} & \mathbf{0} \\ \boldsymbol{\xi} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\omega} \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \mathbf{v} \\ \mathbf{w} \end{bmatrix}. \tag{6}$$

Strains can be computed from Eqs. (1)–(4) by means of the three-dimensional compatibility equations yielding $\varepsilon_{nn} = 0$, $\gamma_{sn} = 0$ and:

$$\boldsymbol{\varepsilon}(n,s,z,t) = \mathbf{E}(s,n)\mathbf{e}(z,t), \tag{7}$$

where

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{ss} \\ \varepsilon_{zz} \\ \gamma_{zs} \\ \gamma_{zn} \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} \mathring{\boldsymbol{\xi}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\omega} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -2n\mathring{\boldsymbol{\psi}} + \mathring{\boldsymbol{\varphi}} + \boldsymbol{\mu} & \frac{1}{2}(\boldsymbol{\mu} - \mathring{\boldsymbol{\varphi}}) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\psi} \end{bmatrix} \tag{8}$$

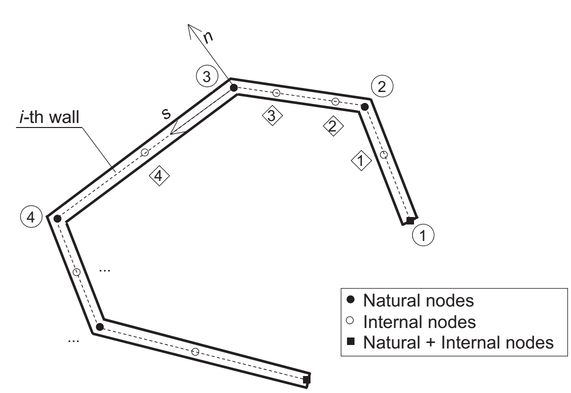

Fig. 1. Thin-walled cross-section.