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Size dependent response of large shell elements under in-plane tensile loading

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ABSTRACT

Large-scale thin-walled structures with a low weight-to-stiffness ratio provide the means for cost and energy efficiency in structural design. However, the design of such structures for crash and impact resistance requires reliable FE simulations. Large shell elements are used in those simulations. Simulations require the knowledge of the true stress–strain response of the material until fracture initiation. Because of the size effects, local material relation determined with experiments is not applicable to large shell elements. Therefore, a numerical method is outlined to determine the effect of element size on the macroscopic response of large structural shell elements until fracture initiation. Macroscopic response is determined by introducing averaging unit into the numerical model over which volume averaged equivalent stress and plastic strain are evaluated. Three different stress states are considered in this investigation: uniaxial, plane strain and equi-biaxial tension. The results demonstrate that fracture strain is highly sensitive to size effects in uniaxial tension whereas in plane strain or equi-biaxial tension size effects are much weaker. In uniaxial and plane strain tension the fracture strain for large shell elements approaches the Swift diffuse necking condition.

1. Introduction

Thin-walled structures with a low weight-to-stiffness ratio provide the means for cost- and energy efficiency in structural design. In the quest for efficiency, the structural safety of lightweight shell structures has become more important as a result of the increased societal awareness regarding accidents and structural failure. This has led to designs that rely on FE simulations as full-scale experiments of such events are impossible to conduct. Simulations involving impact, crush and crashworthiness however, require the knowledge of true stress–strain behavior of the material until fracture initiation.

Recent experimental–numerical studies, e.g. (Zhang et al., 1999; Gruben et al., 2012; Dunand and Mohr, 2010; Tardif and Kyriakides, 2012; Ghahremaninezhad and Ravi-Chandar, 2012), clearly show that an accurate stress–strain response and equivalent plastic strain to fracture initiation $\bar{\varepsilon}^f$, i.e. the fracture strain, are a pre-condition for adequate FE solution in problems involving strain localization and ductile fracture. For the sake of brevity, notation "fracture strain" is used throughout the paper interchangeably with the term "fracture initiation strain". In these studies the stress–strain relation and the fracture strain are determined using a certain experimental length scale. This experimental length scale defines the element size used in the simulations; see e.g. (Hogström et al., 2009; Ehlers and Varsta, 2009). In other words, the FE solution is mesh size sensitive, which accuracy depends on the chosen fracture strain. In the failure analysis of materials and structures, such size effects are an important issue (Bazant, 2000; Fleck and Hutchinson, 1993). In large-scale structural analysis, for practical reasons, the mesh size is usually several orders of magnitude higher than the experimental length scale. For instance, the recommended element aspect ratio in the analysis of large structures is $L_e/t > 5$, where L_e is the element length and t is the plate thickness (Hogström and Ringsberg, 2012). In contrast, the aspect ratio corresponding to the experimental length scale is usually less than 1. Hence, the consistency between the experimental length scale and FE mesh size is lost.

The engineering approach to bridging the two scales is the most intuitive. By introduction of "virtual extensometer", which represents various experimental length scales in a standard tensile test, fracture strain can be determined for larger elements. Stress on the other hand cannot be directly measured, which is why it is calculated based on the minimum cross-sectional area of the specimen independent of the experimental length scale. The true stress–strain curve until fracture obtained this way represents the macroscopic response of large structural shell elements until fracture initiation. Alternatively, for the one-dimensional uniaxial tension case, a

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closed-form analytical expression of the element size to the fracture strain can be derived (Li and Karr, 2009). In any case, the power-law type relationship that relates fracture strain to element size is denoted as Barba's law. Fracture criterion that is based on the critical equivalent plastic strain and is scaled with the Barba's law is referred to as *shear* criterion. This fracture criterion is employed most notably in the analysis of large-scale structural components (Simonsen and Törnqvist, 2004; Alsos et al., 2009; Hogström and Ringsberg, 2013; Ehlers, 2010) and full-size collision and crashworthiness simulations of ship structures, e.g. (Naar et al., 2002; Yamada et al., 2005; Kõrgesaar and Ehlers, 2010; Samuelides, 2012; Hogström and Ringsberg, 2012) to name a few. However, the *shear* criterion is strictly valid only for uniaxial tension. It is known that the fracture ductility is a strong function of the stress state or the stress triaxiality, $\eta = \sigma_h/\bar{\sigma}$, where the hydrostatic and von Mises stress are denoted by σ_h and $\bar{\sigma}$, respectively. Stress state-dependent failure in metals was first observed by McClintock (1968), Rice and Tracey (1969), Hancock and Mackenzie (1976), Gurson (1977) and Johnson and Cook (1985) and in more recent experimental studies, e.g. (Bao and Wierzbicki, 2004; Barsoum and Faleskog, 2007; Haltom et al., 2013; Hopperstad et al., 2003). An example of the influence of triaxiality on the fracture strain in plane stress condition is shown for a steel material in Fig. 1(a). The results of the tensile experiments by Dunand and Mohr (2010) with different notch radii in Fig. 1(b) clearly indicate that the fracture strain is also strongly dependent on the strain path. The importance of stress triaxiality on the fracture strain is also recognized by some of the fracture criteria employed in large-scale structural analysis, namely the Bressan–Williams–Hill (BWH) instability and the Rice–Tracey–Cockcroft–Latham (RTCL) damage criterion described by Alsos et al. (2008) and Törnqvist (2003), respectively. However, the BWH-criterion neglects the size effects completely, as it is argued that size effects appear after local necking. The RTCL criterion is adjusted for different mesh densities based on the fracture strain determined with the uniaxial tension test, i.e., with a Barba's law. Walters (2013) has proposed adjusting the fracture strain on both the mesh size and stress state. However, evidence for such an adjustment is still lacking as no experimental or numerical results were presented.

To fill this gap, we introduce an alternative numerical approach to bridge the local and global scale and thereby describe the size effects at different stress states. The numerical stress–strain response until fracture in global scale, referred to as macroscopic response, is obtained as the volume averaged stress–strain response of a finite averaging unit (AU) that is introduced into the numerical model or specimen. The specimen is imposed to stress states corresponding to multi-axial tension condition: uniaxial tension (UAT, $\eta = 1/3$), plane strain tension (PST $\eta = 1/\sqrt{3}$) and equi-biaxial tension (EBT, $\eta = 2/\sqrt{3}$). Size effects due to the bending are not considered in the present study. Thereby, the combined effect of size and stress state on the fracture strain is established. Size of the averaging unit corresponds to the large structural shell elements used in the analysis of large structures such as ships. Hence, the approach described is fundamentally an extension of the engineering approach used to determine Barba's law from tensile tests to multi-axial stress states. In contrast to engineering approach described above, "averaging unit" introduced to numerical model facilitates the comprehensive analysis of all the field quantities, including the stress and strain state, and their influence on the true stress–strain response and the fracture strain.

2. Approach

2.1. Necking instability

In general, a ductile fracture in sheet metal is preceded by a loss of stability, (Marciniak and Kuczyński, 1967; Hutchinson and Neale, 1979; Xue, 2010). The loss of stability reveals itself during the deformation process as high strain and stress gradients appear over a limited region of the sheet, while in the exterior zones some unloading and softening can take place. This type of plastic flow localization is responsible for the size effects investigated in this study. Depending on the stress state, the intensity of the plastic flow and the size of the localization zone vary. Thereby, the intensity of the size effects in different stress states can vary as well. Specifically, we consider two types of instabilities: diffuse and localized necking. Diffuse necking, which is characteristic of flat tensile specimens and uniaxial tension ($\eta = 1/3$), takes place over the width of the gage section as shown in Fig. 2(a). The amount of diffuse necking is here quantified as the width ratio at the end of the gage section (w_1) vs. the width in the middle section (w_2). Diffuse necking is followed by localized necking, or severe thinning in the middle of the gage section. In metals the width of the local neck is roughly equal to the thickness of the sheet (Hu et al., 2002). The amount of thinning is quantified with the thickness ratio of t_1/t_2 as shown in Fig. 2(a)–(c). This ratio starts to increase already in the diffuse necking stage, but the localized necking triggers a steep growth of the ratio. Therefore, the thickness ratio is associated with the developing local neck. In plane strain ($\eta = 1/\sqrt{3}$) and equi-biaxial tension ($\eta = 2/3$), geometric constraints obviate the diffuse necking, meaning that only localized necks appear in the thickness direction. Geometric constraint in plane strain stems from the boundary conditions, which restrict the plate edges from pulling in as shown in Fig. 2(b), and in equi-biaxial tension from the loading as shown in Fig. 2(c), which

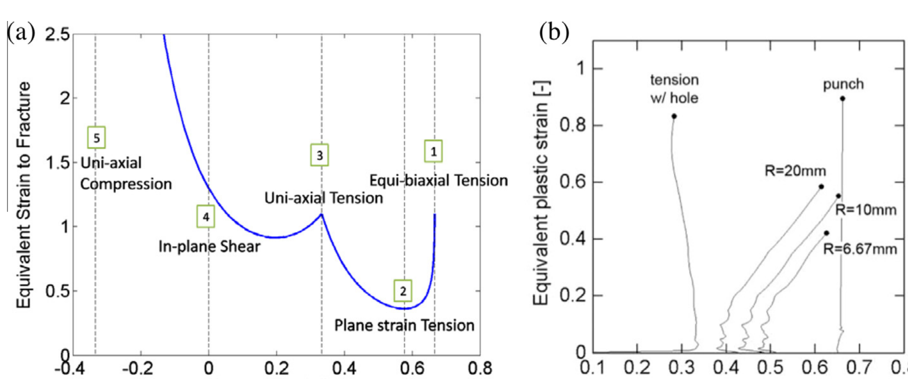

Fig. 1. Influence of stress triaxiality on fracture ductility for two advanced high-strength steels. (a) Fracture envelope from Luo and Wierzbicki (2010) and (b) triaxiality history until fracture initiation for three different specimens (Dunand and Mohr, 2010).