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Constitutive modelling of the amplitude and frequency dependency of filled elastomers utilizing a modified Boundary Surface Model

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ABSTRACT

A phenomenological uniaxial model is derived for implementation in the time domain, which captures the amplitude and frequency dependency of filled elastomers. Motivated by the experimental observation that the frequency dependency is stronger for smaller strain amplitudes than for large ones, a novel material model is presented. It utilizes a split of deformation between a generalized Maxwell chain in series with a bounding surface plasticity model with a vanishing elastic region. Many attempts to capture the behaviour of filled elastomers are found in the literature, which often utilize an additive split between an elastic and a history dependent element, in parallel. Even though some models capture the storage and loss modulus during sinusoidal excitations, they often fail to do so for more complex load histories. Simulations with the derived model are compared to measurements in simple shear on a compound of carbon black filled natural rubber used in driveline isolators in the heavy truck industry. The storage and loss modulus from simulations agree very well with measurements, using only 7 material parameters to capture 2 decades of strain (0.5–50% shear strain) and frequency (0.2–20 Hz). More importantly, with material parameters extracted from the measured storage and loss modulus, measurements of a dual sine excitation are well replicated. This enables realistic operating conditions to be simulated early in the development process, before an actual prototype is available for testing, since the loads in real life operating conditions frequently are a combination of many harmonics.

1. Introduction

Rising fuel prices, an increased environmental awareness and legal demands are incentives for the automotive industry to reduce fuel consumption, which efficiently lowers both costs and emissions. This reduction is achieved by for instance, lower revolutions at cruising speed, start-stop functionality, fuel cells, hybrid engines and lighter vehicles. At the same time, the demands on noise, vibration and harshness characteristics are ever increasing. Driveline isolators and rubber bushings have to be designed to fulfil these demands regardless of which driveline is used. In order to reduce cost and development time, computer aided simulations are used in the design process. However, the results of such simulations are, at most, only as good as the material models employed.

Almost all bushings, vibration isolators and shock absorbers in automotive vehicles are made out of elastomers. Elastomers' unique properties of high extensibility, damping and the introduction of a significant change in impedance make it ideal for the task of isolating the vibrations from the combustion engine and uneven roads (Sjöberg and Kari, 2002). However, in order to achieve sufficient stiffness, tear strength and an increased fatigue resistance, a reinforcing filler is often used (Heinrich et al., 2002).

For temperatures above the glass transition temperature unfilled elastomers have a stress–strain behaviour that for moderate, quasi-static, strains can be described by statistical and continuum mechanics (Boyce and Arruda, 2000; Treloar, 2005; Edwards and Vilgis, 1998). The addition of a reinforcing filler to the material introduces a strong strain amplitude dependency commonly referred to as the Fletcher–Gent effect, which is significant for strains larger than 0.01–0.1% (Fletcher and Gent, 1953; Payne and part, 1962; Kraus, 1984; Rendek and Lion, 2010; Wrana and Härtel, 2008). Therefore, the classical material models for unfilled elastomers are inadequate. Even more cumbersome, the response

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of a filled elastomer subjected to two simultaneous sinusoidal displacements does not equal the sum of the responses of the individual excitations (Wrana and Härtel, 2008; Sjöberg and Kari, 2003). Consequently, it is not an easy task to formulate an accurate material model in the frequency domain or calculate the response using Fourier methods since the superposition principle is not valid. Therefore, it is beneficial to formulate constitutive equations in the time domain. Furthermore, the vulcanization process and thereby the rubber compound are often undisclosed which means that the information needed for determining material parameters from the physical structure of the material would generally be unavailable. Hence, a phenomenological approach is advantageous when modelling filled elastomers with material parameters that can be deduced from small test samples and simple experiments.

A method frequently used to model filled elastomers is to additively decompose it into an elastic part and a history-dependent part, in parallel. The simplest model that combines relaxation with a stiffness at quasi-static loading conditions is the well known standard linear solid (SLS). However, measurements have shown that vulcanized filled elastomers contain a spectrum of relaxation times (Adolfsson et al., 2005; Lion, 1998) where the SLS for obvious reasons is limited to a single relaxation time. For a finite range of frequencies, this can be modelled by adding more Maxwell elements in parallel to the SLS, resulting in a generalized Maxwell chain (GMC). Another way of capturing a spectrum of relaxation times is with fractional derivatives (Adolfsson et al., 2005). The major drawback of fractional derivatives is that the results from all previous time steps are needed for the calculation of the next, which makes it computationally expensive. Even though this to some extent has been solved by implementing a sparse time history (Adolfsson et al., 2004; Adolfsson, 2004), neither fractional derivatives nor the conventional GMC have the ability to capture the observed amplitude dependency of filled elastomers, without modifications.

By utilizing non-linear viscoelastic models with process dependent relaxation times an amplitude dependency can be introduced (Rendek and Lion, 2010; Höfer and Lion, 2009; Liu and Fatt, 2011). However, choosing evolution laws for the internal variables is often a complicated matter with many material parameters to define, which often leads to a trade-off if the storage or the loss modulus should be represented accurately.

Another technique often used to model the Fletcher–Gent effect is with plastic elements. For uniaxial loading conditions, many models exist that capture the smooth plastic behaviour of filled elastomers, one being the standard triboelastic solid (Coveney et al., 1995; Coveney and Johnson, 1999). The strength of this and similar models (Berg, 1995; Dahl et al., 1960; Netzker et al., 2010) is that the amplitude dependency of the complex modulus can be well reproduced, but their main drawback is the absence of frequency dependency. This problem is addressed either by adding a strain rate dependency in the plastic element (Coveney and Johnson, 2000; Hu and Wereley, 2012) or by adding for instance a fractional derivative (Sjöberg and Kari, 2002; García Tárrago et al., 2007) or a GMC (Yarmohamadi and Berbyuk, 2010; Gracia et al., 2010) in parallel to the elastoplastic model.

For the finite element method, there exist phenomenological constitutive equations implemented for three-dimensional analysis that capture the amplitude dependency and hysteresis in filled elastomers for large deformations, such as the MORPH-model (Besdo and Ihlemann, 2003) or an endochronic plasticity formulation (Netzker et al., 2010). Unfortunately these material models have no strain rate dependency, but this can be improved by using an overlay method (Gracia et al., 2010) and a viscoelastic model suitable for large deformations (Govindjee and Reese, 1997; Bergström and Boyce, 1998).

However, measurements in literature clearly show that when the frequency is increased, the increase in stiffness in absolute values is larger for smaller strain amplitudes (Sjöberg and Kari, 2002; Rendek and Lion, 2010; Lion and Kardelky, 2004; Chazeu et al., 2000; Luo et al., 2010). This is a strong indication that an additive split between plastic and viscous elements is not an accurate approach in a material model with the ambition to capture the response in filled elastomers. From a physical point of view, it is acknowledged that the Fletcher–Gent effect is caused not solely from the breakdown and reforming of filler-filler structures, but that there is a substantial contribution from filler-polymer interactions (Ahmadi and Muhr, 2011; Donnet and Custodero, 2013; Fröhlich et al., 2005; Litvinov et al., 2011). This is interpreted as polymer chains in series with a mechanism which causes energy dissipation, which again suggests that a polymer network in series with a plastic contribution is suitable for modelling filled elastomers.

In this paper, a constitutive model for filled elastomers is presented, with the novelty being an addition of a frequency dependency to the bounding surface model with a vanishing elastic region (Dafalias and Popov, 1977). The derived model will be compared to measurements on a compound of carbon black filled natural rubber, used as isolators in the heavy truck industry. Finally, the model proposed is intended for the evaluation of the dynamical behaviour of rubber compounds with reinforcing fillers. If the stresses and strains are reproduced well, the durability of a component and the surrounding structure has the potential to be optimized. For many load bearing components made out of rubber, the strains are typically moderate to large (50% shear strain is not uncommon) and in the lower frequency domain (typically below 20 Hz). This calls for a material model that can handle large perturbations from an equilibrium condition. However, it needs only to be accurate in a relatively small frequency bandwidth (0.2–20 Hz). Also, in the intended application, the strains are not expected to be large enough for the finite extensibility of the polymer chains to influence the dynamical stiffness of the components. Such high strain amplitudes could result in an unacceptably low durability (Sheridan et al., 2001).

The presented model enables the properties of components subjected to uniaxial loading conditions to be studied via simulations before an actual prototype is available for testing, given that small test specimens to determine material parameters are often accessible early in the design process.

2. Measurements

The measurements are conducted in a GABO EPLEXOR® 500 N, fitted with a force transducer with a measurement range of 150 N. The experimental setup and the dimensions of the rubber specimen are shown in Fig. 1. The rubber was injection moulded between steel parts, the two outward pieces were attached to a baseplate and a displacement is applied via the middle piece,

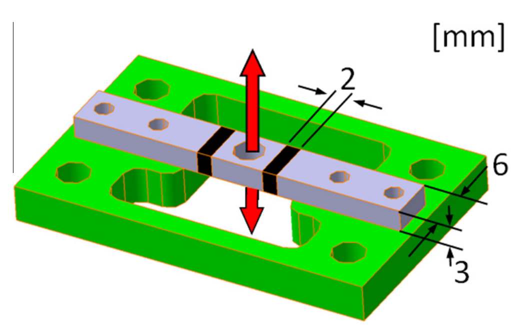

Fig. 1. Test specimen. Rubber compound vulcanized between three steel pieces. Displacement applied on middle piece.