International Journal of Solids and Structures 51 (2014) 3471–3479

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Modeling photo-induced deformation of glassy splay-bend and twist nematic sheets

L.H. He [a,*], D.W. Huang [a,b]

[a] CAS Key Laboratory of Mechanical Behavior and Design of Materials, University of Science and Technology of China, Hefei, Anhui 230026, PR China
[b] College of Architectural and Civil Engineering, Jiaxing University, Jiaxing, Zhejiang 314001, PR China

ARTICLE INFO

Article history:
Received 14 November 2013
Received in revised form 26 May 2014
Available online 28 June 2014

Keywords:
Glassy nematic sheet
Splay-bend
Twist
Deformation

ABSTRACT

Azobenzene-containing glassy nematic sheets deform in response to light in a complicated way depending on director distribution. To quantify the large-deflected deformation, a theoretical model is developed for the sheets with typical splay-bend and twist director distributions. A third-order in-plane displacement assumption is adopted to characterize the effect of transverse shear deformation, and the necessity is discussed through two examples for which analytical solutions are obtainable. Though this work is an extension of the third-order shear deformable theory for anisotropic laminates, it involves some new ingredients such as varying spontaneous strains and special material symmetries. The results are expected useful for analysis and design of the glassy nematic sheets in actuation applications.

1. Introduction

Nematic glasses and elastomers with azobenzene moieties deform quickly and reversibly in response to low intensity UV light (Finkelmann et al., 2001; Warner and Terentjev, 2003; Yu et al., 2003; Zhao and Ikeda, 2009). The phenomenon originates from photo-isomerization of the azobenzene moieties which, upon irradiation, are transferred from the elongated *trans*-state to the strongly bent *cis*-state. This disturbs the nematic order and results in spontaneous contraction and expansion along and perpendicular to the molecular director, respectively. Blue–green light has also been used to deform the materials (Lee et al., 2012), but the mechanism for deformation is a *trans–cis–trans* isomerization, including some reorientation of the *trans* isomers. The unique photo-responsivity provides an efficient way of transferring energy to the working element without the need for electrodes and wires, and thus makes the nematic solids extremely attractive for actuation applications, especially in the situation when remote addressing is desired (Camacho-Lopez et al., 2004; Lee et al., 2011; Modes et al., 2013; van Oosten et al., 2009).

We are concerned in this work with nematic glasses. Due to the high cross-linking density, chain motion in these materials is highly limited so that they have high elastic moduli in the gigapascal range (Mol et al., 2005; van Oosten et al., 2007). Though the spontaneous strains are only a few percent, the director is not independently mobile from the elastic matrix as it is in elastomers and thus is controllable. Indeed, glassy nematic sheets with various director distributions can be fabricated (de Haan et al., 2012; van Oosten et al., 2007), allowing the photomechanical responses to be well engineered. Splay-bend and twist distributions are most typical. The architectures are shown in Fig. 1a and b, referring a global coordinate system (x_1, x_2, x_3) whose $x_1 - x_2$ plane coinciding with the mid-plane of the sheet, and a local system (x'_1, x'_2, x'_3) fixed to the director with the x'_3 axis along the director. In the splay-bend sheet, the director orientation changes continuously in the $x_1 - x_3$ plane from along the x_1 direction on the upper to along the x_3 direction on the lower surface, making an angle $\phi = \pi(1 - 2x_3/h)/4$ with the x_1 axis at a point of thickness coordinate x_3. In the twist sheet, the director rotates smoothly about the x_3 axis on traversing the thickness and alters the orientation from parallel to the x_1 direction on the top to parallel to the x_2 direction on the bottom surface, also making an angle $\phi = \pi(1 - 2x_3/h)/4$ with the x_1 axis. These director fields lead to gradient mechanical responses of the sheets across the thickness, so that bend can occur even if the photoexcitation is uniform.

Some efforts have been made to explore the influence of the complex director distributions on photo-induced deformation of the nematic sheets. Based on elastic isotropy and small deformation, Warner et al. (2010) derived curvatures of free-standing splay-bend and twist beams. The results indicate that a splay-bend beam bends only in one direction upon illumination while a twist

* Corresponding author. Tel.: +86 551 63603182.
E-mail address: lhhe@ustc.edu.cn (L.H. He).

http://dx.doi.org/10.1016/j.ijsolstr.2014.06.016
0020-7683/

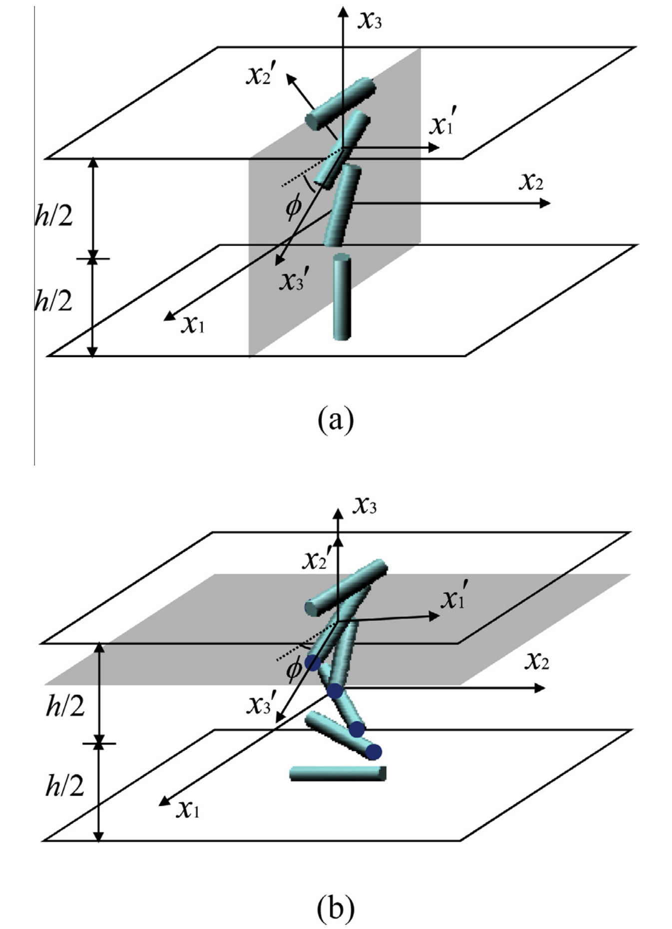

Fig. 1. Two typical director distributions in a glassy nematic sheet: (a) splay-bend, and (b) twist.

one bends in two opposite directions to form a saddle. Modes et al. (2010) further examined the effect of elastic anisotropy and found that it plays no qualitative role in affecting the weak response of the nematic beams with anisotropic spontaneous strains. Recently, He (2013) developed a three-dimensional model to predict small deformations of constrained splay-bend and twist sheets based on isotropic elasticity. Application of the model to free-standing nematic sheets indicated that the results given by Warner et al. (2010) are universal in the sense that they are valid for arbitrary geometries. Smith et al. (2014) proposed a theoretical framework for nematic beams under irradiation of polarized light. An interesting design of light driven flexural–torsional deformation was also illustrated with experimental confirmation. Despite these progresses, however, a comprehensive model which accounts for the collaborative effects of director distribution, geometric shape, external load and large deflection of a nematic sheet is still lacking. Indeed, varying director orientation causes elastic anisotropy and inhomogeneity and thus results in complicated deformation and stress distribution in a nematic solid, but the current knowledge about this is limited only to free-standing sheets (Modes et al., 2010). An external normal load and a special director distribution (e.g. splay-bend) both induce transverse shearing, the combined effect determines the bending behavior of a nematic sheet and therefore must be well characterized in theoretical modeling. Moreover, it is common for nematic sheets with large span-to-thickness ratio that the deflection is equal to or larger than the thickness. In this situation the linear elasticity theory ceases to work, and geometric nonlinearity has to be incorporated into the analysis.

Motivated by the above reason, we propose here a general model for photo-induced deformation of glassy splay-bend and twist nematic sheets. In comparison with our previous work (He, 2013), the present study takes into account not only the elastic anisotropy and inhomogeneity, but also arbitrary normal load and geometric nonlinearity. The paper is organized as follows. In the next section, the anisotropic spontaneous strains and stress–strain relations of the two types of nematic sheets are given. A displacement expression which satisfies the equilibrium condition of transverse shear stress on the top and bottom surfaces of the sheet, as in the third-order plate theory (Reddy, 1984a, b), is constructed in Section 3. In Section 4, the geometrically nonlinear governing equations and the associated boundary conditions of the sheets are derived via the principle of virtual work. Two illustrative examples for the photo-induced deformation of cantilevered splay-bend and twist strip-like sheets are analyzed and discussed in Section 5, followed by a brief conclusion in Section 6. Throughout this paper, without special elucidation, a Latin subscript runs from 1 to 3 while a Greek one takes values of 1 or 2, with repeated subscripts meaning summation. In addition, a comma stands for differentiation with suffix coordinate.

2. Spontaneous strains and stress–strain relations

We start with considering an arbitrarily shaped nematic sheet of thickness h, illuminated on the upper surface by a perpendicular incident UV light of intensity I_0. The director distribution may be either splay-bend or twist. To describe the photomechanical response of the entire sheet, we first give the local spontaneous strains and stress–strain relation with respect to the local coordinate system (x_1', x_2', x_3'). The global spontaneous strain distributions and stress–strain relations referring the (x_1, x_2, x_3) system then are obtained separately for the splay-bend and twist director conformations by coordinate transformation.

2.1. Local photomechanical response

The illumination not only causes spontaneous contraction and expansion along and normal to the director, but also gives rise to temperature change relative to the non-illuminated state. Since the nematic sheet is very thin (typically 10 μm), it is assumed that the temperature change is uniform over the sheet. The adsorption of the azobenzene moieties in the photo-stationary UV-illuminated state is isotropic, and the light intensity attenuates exponentially with the penetration depth according to Beer's law. Therefore, when the intensity I_0 is low, the local spontaneous strains ε'_{ij} at a point (x_1, x_2, x_3) in a splay-bend or twist sheet can be written system as (Corbett and Warner, 2006; van Oosten et al., 2007)

$$\begin{aligned} \varepsilon'_{11} = \varepsilon'_{22} = \varepsilon_\perp = \varphi P_\perp I_0 e^{-\varphi x_3/d} + \alpha_\perp \Delta T, \\ \varepsilon'_{33} = \varepsilon_\parallel = \varphi P_\parallel I_0 e^{-\varphi x_3/d} + \alpha_\parallel \Delta T, \end{aligned} \tag{1}$$

with respect to the (x_1', x_2', x_3'). Here the symbols "$\parallel$" and "$\perp$" denote effects along and perpendicular to the director, respectively, φ is the fraction of azobenzene monomer, P stands for the photocompliance, α is the thermal expansion coefficient, and d means the attenuation length per fraction azobenzene.

The spontaneous strains result in the deformation of the sheet. For a complete characterization, five independent elastic constants are required: two Young's moduli $E_\parallel$ and $E_\perp$, two Poisson's ratios $\nu_\parallel$ and $\nu_\perp$, and one shear modulus G. In the local coordinate system (x_1', x_2', x_3'), the relationship between the stress σ'_{ij} and strain ε'_{ij} is expressed by (Modes et al., 2010)