International Journal of Solids and Structures 51 (2014) 3480–3489

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Regularization of microplane damage models using an implicit gradient enhancement

Imadeddin Zreid, Michael Kaliske *

Institute for Structural Analysis, Technische Universität Dresden, 01062 Dresden, Germany

ARTICLE INFO

Article history:
Received 4 April 2014
Received in revised form 28 May 2014
Available online 1 July 2014

Keywords:
Microplane model
Damage
Implicit gradient enhancement
Concrete

ABSTRACT

The microplane model allows for the description of damage induced anisotropy in a natural manner by introducing constitutive laws for quantities on individual microplanes at each material point. However, if damage or other strain softening constitutive laws are used within the microplane approach, the well-known problem of localization arises leading to spurious results and mesh dependency. This problem demands some regularization method to stabilize the solution. The paper focuses on the efficient implementation of implicit gradient enhancement for microplane damage models. Previous works enhanced the strain tensor, thus resulting in large number of extra degrees of freedom, which limits the use of this method for large scale 3D simulations. A new method which enhances the equivalent strain driving the damage on each microplane is introduced in this work. The new method limits the number of additional degrees of freedom to one, while preserving the regularizing effect. The two methods are implemented within a 3D finite element code to compare their performance. The microplane model used is based on a thermodynamically consistent formulation and on a volumetric–deviatoric split of strains on each microplane. Furthermore, an exponential damage law is used and an equivalent strain expression which distinguishes between compression and tension is applied to simulate the behavior of concrete. The capabilities of the proposed formulation are demonstrated by comparison to published experiments on plain concrete.

1. Introduction

The microplane model is a powerful tool for modeling concrete and other quasi-brittle materials (Bažant et al., 2000; Caner and Bažant, 2013a). The behavior of those materials is characterized by the transition from isotropic to anisotropic response once the material enters the inelastic regime. Concrete, for example, which consists of different constituents, exhibits upon loading initiation of microcracks often at the interfaces between aggregates and the mortar matrix. The growth of these microcracks leads to anisotropic behavior and eventually to macroscopic cracks and failure. The microplane approach provides a simple and straightforward way to model this phenomenon by defining the constitutive material relations between stress and strain vectors on randomly oriented planes. Since the pioneering work by Bažant and Prat (1988), it has been researched extensively, and a variety of constitutive material laws has been implemented within the microplane approach including damage and plasticity. However, the strain localization problem which is well known for strain softening constitutive models persists also with microplane models. This problem is caused by ill-posedness of the governing differential equations in case of strain softening material laws, which leads to pathological mesh dependency and numerical instability of the finite element solution.

Many remedies have been proposed to counter this problem and some of them have been already used to regularize microplane damage models. One powerful and physically motivated method is the nonlocal integral type approach. Although usually motivated by its ability to eliminate mesh dependency and slow convergence rate, micromechanical arguments have been also presented. For example, the dependency of damage at one microcrack on the release of stored energy from its neighborhood, and the effect of material inhomogeneity, which causes the dependency of the stress state at a given material point on its surrounding region (Bažant, 1991). One difficulty with the integral type formulation is that it leads to a set of integro-differential equations, which require sharing information between points, thus abandoning the advantage of classical finite element method and complicating

* Corresponding author. Tel.: +49 351 463 34386.
E-mail address: michael.kaliske@tu-dresden.de (M. Kaliske).

http://dx.doi.org/10.1016/j.ijsolstr.2014.06.020
0020-7683/

the implementation within the finite element software. This issue motivated the so-called gradient enhanced models, which preserve the mathematical locality of the finite element method, while taking the field around the point into account by enhancing the equations with higher gradients of strain or other internal variables. There are two types of gradient models, explicit and implicit. Explicit gradient enhancement is only weakly nonlocal, thus fails to regularize the solution under some circumstances. Implicit gradient enhancement, on the other hand, have the advantage of being strongly nonlocal and largely equivalent to the integral type (Peerlings et al., 2001), while keeping the differential nature of the equations, thus results in a straightforward implementation in finite element codes. Its idea is to introduce a second differential equation to calculate the nonlocal field, which is usually the counterpart of the local strain or other local internal variables. The expense to be paid in this case is adding extra degrees of freedom. For isotropic damage models, it is sufficient to apply the enhancement for a scalar valued quantity, such as the equivalent strain (Saroukhani et al., 2013), thus the extra degrees of freedom is limited to 1. This fact makes implicit gradient enhancement for isotropic damage models very successful. On the other hand, for anisotropic models, the nonlocal field needs to be a tensorial quantity, or in case of microplane models a scalar quantity at every microplane. This problem means that the number of extra degrees of freedom is very large and renders the method unpractical for large scale 3D simulations.

Though the microplane model has been researched extensively, little attention has been paid to the regularizing techniques which are essential for the practical application of the model (Caner and Bažant, 2013a). A nonlocal integral type method has been implemented for the microplane model, for instance, in Bažant and Di Luzio (2004), Bažant and Ozbolt (1990) and Luzio (2007). Implicit gradient enhanced microplane models have been introduced in Kuhl et al. (2000) and Leukart (2005), where the strain tensor has been used as the nonlocal variable. The aim of this work is to explore the feasibility of formulating an efficient and reliable way to regularize the microplane damage models using an implicit gradient enhancement. The paper is organized as follows. Firstly, the gradient model used in Kuhl et al. (2000) and Leukart (2005) is reviewed and explained. Afterwards, a new simplified method for the implicit gradient enhancement is derived. Finally, the behavior of the two methods is demonstrated and compared by simulations of experiments on plain concrete.

2. Strain gradient model

2.1. Finite element formulation

The gradient enhanced microplane damage model in Kuhl et al. (2000) and Leukart (2005) is based on enhancing the strain tensor. This means that a tensorial nonlocal field is considered. The system is then governed by 2 coupled differential equations and solved using a simultaneous, fully coupled scheme. The first equation is balance of linear momentum for quasi-static case

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{f} = 0 \tag{1}$$

and the second is the modified Helmholtz equation to describe the nonlocal strain tensor

$$\bar{\boldsymbol{\epsilon}} - c\nabla^2 \bar{\boldsymbol{\epsilon}} = \boldsymbol{\epsilon}, \tag{2}$$

with the homogenous Neumann boundary condition

$$\nabla \bar{\boldsymbol{\epsilon}} \cdot \boldsymbol{n}_b = \mathbf{0}, \tag{3}$$

where, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\nabla\cdot$ is the divergence operator and $\boldsymbol{f}$ is the body force vector. Moreover, $\boldsymbol{\epsilon}$ is the local strain tensor, $\bar{\boldsymbol{\epsilon}}$ is its nonlocal counterpart, c is the gradient activity parameter, $\boldsymbol{n}_b$ is the normal to the outer boundary, ∇ is the gradient operator and ∇^2 is the Laplace operator.

The homogenous Neumann boundary condition adopted here is commonly used and it is enough to ensure the regularizing effect. With this boundary condition the local and nonlocal strains are equal for homogenous deformations and the gradient method is, therefore, consistent with integral type formulation. Peerlings et al. (2001) showed that this type of boundary condition provides larger nonlocal weight factors for the material close to the external boundaries. This is motivated from the physical point of view, because the model in this case will be more sensitive to surface effects. This boundary condition is applied to the entire external boundary regardless whether there are applied displacements or loads to some regions or not, since a physical connection between the two fields boundaries is not clear.

To get the weak form of Eqs. (1) and (2), they are multiplied by the weight functions $\delta\mathbf{u}$ and $\delta\bar{\boldsymbol{\epsilon}}$, respectively,

$$\int_{\mathcal{B}} \delta\mathbf{u} \cdot \nabla \cdot \boldsymbol{\sigma} dv + \int_{\mathcal{B}} \delta\mathbf{u} \cdot \boldsymbol{f} dv = 0, \tag{4}$$

$$\int_{\mathcal{B}} \delta\bar{\boldsymbol{\epsilon}} \cdot \bar{\boldsymbol{\epsilon}} dv - \int_{\mathcal{B}} \delta\bar{\boldsymbol{\epsilon}} \cdot c\nabla^2 \bar{\boldsymbol{\epsilon}} dv = \int_{\mathcal{B}} \delta\bar{\boldsymbol{\epsilon}} \cdot \boldsymbol{\epsilon} dv. \tag{5}$$

Substituting the relation $\nabla \cdot (\boldsymbol{\sigma} \cdot \delta\mathbf{u}) = \delta\mathbf{u} \cdot \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} : \nabla\delta\mathbf{u}$, Gauss divergence theorem $\int_{\partial\mathcal{B}} \boldsymbol{\sigma}\boldsymbol{n}_b \cdot \delta\mathbf{u} da = \int_{\mathcal{B}} \nabla \cdot (\boldsymbol{\sigma} \cdot \delta\mathbf{u}) dv$ and Cauchy theorem $\boldsymbol{\sigma} \cdot \boldsymbol{n}_b = \mathbf{t}_e$ in Eq. (4) yield

$$\int_{\partial\mathcal{B}} \mathbf{t}_e \cdot \delta\mathbf{u} da - \int_{\mathcal{B}} \boldsymbol{\sigma} : \nabla\delta\mathbf{u} dv = \int_{\mathcal{B}} \delta\mathbf{u} \cdot \boldsymbol{f} dv \tag{6}$$

and similarly for Eq. (5), substituting the relation $\nabla \cdot (\delta\bar{\boldsymbol{\epsilon}} \cdot \nabla\bar{\boldsymbol{\epsilon}}) = \delta\bar{\boldsymbol{\epsilon}} \cdot \nabla^2\bar{\boldsymbol{\epsilon}} + \nabla\delta\bar{\boldsymbol{\epsilon}} \cdot \nabla\bar{\boldsymbol{\epsilon}}$, Gauss divergence theorem $\int_{\partial\mathcal{B}} \nabla\bar{\boldsymbol{\epsilon}} \cdot \boldsymbol{n}_b \cdot \delta\bar{\boldsymbol{\epsilon}} da = \int_{\mathcal{B}} \nabla \cdot (\delta\bar{\boldsymbol{\epsilon}} \cdot \nabla\bar{\boldsymbol{\epsilon}}) dv$ and the boundary condition $\nabla\bar{\boldsymbol{\epsilon}} \cdot \boldsymbol{n}_b = 0$ yield

$$\int_{\mathcal{B}} \delta\bar{\boldsymbol{\epsilon}} \cdot \bar{\boldsymbol{\epsilon}} dv + \int_{\mathcal{B}} \nabla\delta\bar{\boldsymbol{\epsilon}} \cdot c\nabla\bar{\boldsymbol{\epsilon}} dv = \int_{\mathcal{B}} \delta\bar{\boldsymbol{\epsilon}} \cdot \boldsymbol{\epsilon} dv. \tag{7}$$

Space discretization for the finite element method is obtained by dividing the domain $\mathcal{B}$ into sub-domains $\mathcal{B}^e \subset \mathcal{B}$. Interpolation within the elements is achieved with eight nodes using linear shape functions $\boldsymbol{N}(\xi,\eta,\zeta)$ within the isoparametric concept of finite element method, where ξ, η and ζ are local coordinates that can have values from -1 to 1. Then, the displacement field, and the variational field $\delta\mathbf{u}$ may be interpolated over the sub-domains as follows

$$\mathbf{u} = \boldsymbol{N}(\xi,\eta,\zeta)\boldsymbol{d}^e, \quad \delta\mathbf{u}(\xi,\eta,\zeta) = \boldsymbol{N}(\xi,\eta,\zeta)\delta\boldsymbol{d}^e \tag{8}$$

and the gradient of the displacement field is given as

$$\nabla\mathbf{u} = \partial_{\mathbf{x}}\boldsymbol{N}\boldsymbol{d}^e = \boldsymbol{B}\boldsymbol{d}^e, \quad \nabla\delta\mathbf{u} = \partial_{\mathbf{x}}\boldsymbol{N}\delta\boldsymbol{d}^e = \boldsymbol{B}\delta\boldsymbol{d}^e. \tag{9}$$

Similarly, the nonlocal strain field and its variational field are also interpolated with linear shape functions $\overline{\boldsymbol{N}}$ as follows

$$\bar{\boldsymbol{\epsilon}} = \overline{\boldsymbol{N}}\boldsymbol{E}^e, \quad \delta\bar{\boldsymbol{\epsilon}} = \overline{\boldsymbol{N}}\delta\boldsymbol{E}^e \tag{10}$$

and the gradient of the nonlocal field is given as

$$\nabla\bar{\boldsymbol{\epsilon}} = \partial_{\mathbf{x}}\overline{\boldsymbol{N}}\boldsymbol{E}^e = \overline{\boldsymbol{B}}\boldsymbol{E}^e, \quad \nabla\delta\bar{\boldsymbol{\epsilon}} = \partial_{\mathbf{x}}\overline{\boldsymbol{N}}\delta\boldsymbol{E}^e = \overline{\boldsymbol{B}}\delta\boldsymbol{E}^e, \tag{11}$$

where $\boldsymbol{d}^e$ are the nodal displacements and $\boldsymbol{E}^e$ are the nodal nonlocal strains. The equations have to be satisfied for all admissible $\delta\boldsymbol{d}^e$ and $\delta\boldsymbol{E}^e$, so finally Eqs. (6) and (7) become

$$\int_{\mathcal{B}} \boldsymbol{B}^T\boldsymbol{\sigma} dv = \int_{\mathcal{B}} \boldsymbol{N}^T\boldsymbol{f} dv + \int_{\partial\mathcal{B}_e} \boldsymbol{N}^T\mathbf{t}_e da, \tag{12}$$

$$\int_{\mathcal{B}} \overline{\boldsymbol{N}}^T\bar{\boldsymbol{\epsilon}} dv + \int_{\mathcal{B}} \overline{\boldsymbol{B}}^T c\nabla\bar{\boldsymbol{\epsilon}} dv = \int_{\mathcal{B}} \overline{\boldsymbol{N}}^T\boldsymbol{\epsilon} dv. \tag{13}$$