International Journal of Solids and Structures 51 (2014) 2062–2072

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Analysis of thermo-mechanical behaviour of a crack using XFEM for Leak-before-Break assessments

Peter Gill [a,b,*], Keith Davey [a]

[a] University of Manchester, School of Mechanical, Aerospace, and Civil Engineering, PO Box 88, Manchester M60 1QD, UK
[b] AMEC Clean Energy, Walton House, Birchwood Park, Warrington, Chesire WA3 6GA, UK

ARTICLE INFO

Article history:
Received 19 July 2013
Received in revised form 15 January 2014
Available online 28 February 2014

Keywords:
Thermoelasticity
Crack opening area
Leak-before-Break
XFEM
Thermal stress
Leak rates
Fracture mechanics
Heat transfer

ABSTRACT

The stresses near a crack which has a fluid escaping through it are presented in this paper. The pressure and heat flux, due to the fluid acting on the crack walls, are imposed as boundary conditions in a new finite element tool which has been developed specifically for Leak-before-Break. This special tool uses the extended finite element method to include information about the problem on a sub element level. It is shown to be as accurate as standard finite element models which use very refined meshes, but having the added benefit of being much quicker to implement, and vastly reducing postprocessing. This means that leak rates can be investigated more efficiently. The model is thermo-elastic, and plasticity is accounted for by a correction to the crack opening displacement based on the R6 method. Both crack opening area and peak stresses are shown to decrease when the walls of the crack are hotter than the background plate temperature. The consequences of this for Leak-before-Break assessments are discussed in the paper.

1. Introduction

When carrying out a Leak-before-Break assessment, it is important to obtain an accurate prediction of leak rate. This can be done using fracture mechanics to calculate the crack opening area, then from this a mass flow rate can be deduced, based on the channel depth and inlet conditions. The fracture mechanics and fluid mechanics are generally considered separately (Sharples and Clayton, 1990), this paper demonstrates how the two can be coupled through a new finite element method.

When a fluid which is at high temperature and pressure leaks through a crack, there is an influence on the structure. This is due to the additional heating from the fluid to the structure along the crack walls, as well as the pressure acting to open the crack. These two effects necesitate the use of thermo-mechanical models which can account for the local behaviour at the crack. Analytical models, which use complex stress potentials to solve the case of an insulated crack in a uniform stress field, are well established (Sekine, 1977). However, these are limited to the case of a crack in a plate and more complex cases require numerical analysis.

The finite element method presented here, is designed to capture all the relevant physics when a fluid leaks through a crack, within a single element. This is acheived using the extended finite element method (XFEM), which can embed discontinuities and singular behaviour within elements. In addition to this, special jump terms are added to the approximation space to account for the pressure and heat flux of the leaking fluid acting along the crack. The new element will be referred to as xLbB.

The enrichment of the temperature field is sufficiently general to model discontinuities in temperature along the crack, as well as a discontinuity in the heat flux. The justification of these enrichment terms is based on a convergence study performed using analytical solutions derived via a Green's function method.

This paper is structured as follows: first the background to the problem is considered, where the variational weak form will be stated. The finite element approximation is presented, with a description of all the enrichment functions used, as well as the jump terms. A comparison of crack opening displacement for a hot crack under pressure is presented in Section 4, where the model used for comparison is a focussed mesh in Abaqus using standard elements. The heat flux and stress plots are discussed, as well as the leak rates for Pressurised Water Reactor (PWR) conditions.

* Corresponding author at: University of Manchester, School of Mechanical, Aerospace, and Civil Engineering, PO Box 88, Manchester M60 1QD, UK. Tel.: +44 07894204144.

E-mail address: peter.gill@amec.com (P. Gill).

http://dx.doi.org/10.1016/j.ijsolstr.2014.02.007
0020-7683/

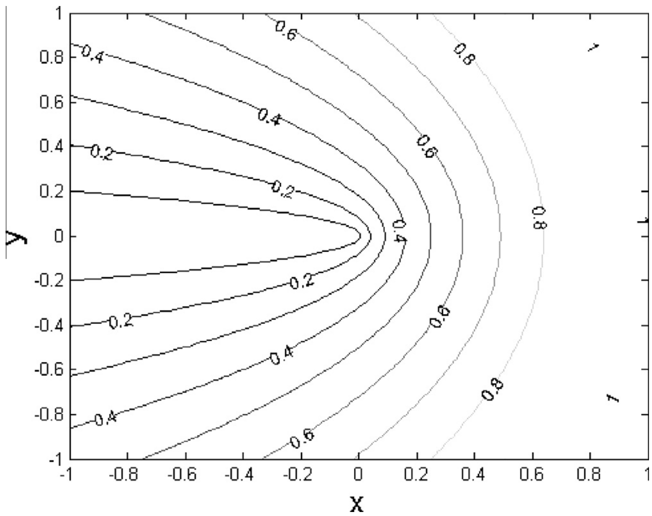

Fig. C.1. Enrichment function $\gamma_1 = \sqrt{r}\cos(\theta/2)$.

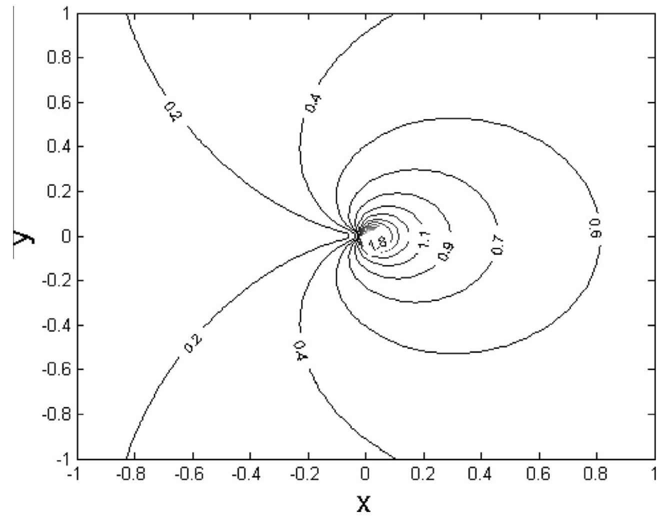

Fig. C.3. Derivative of enrichment function $\frac{d\gamma_2}{dy}$.

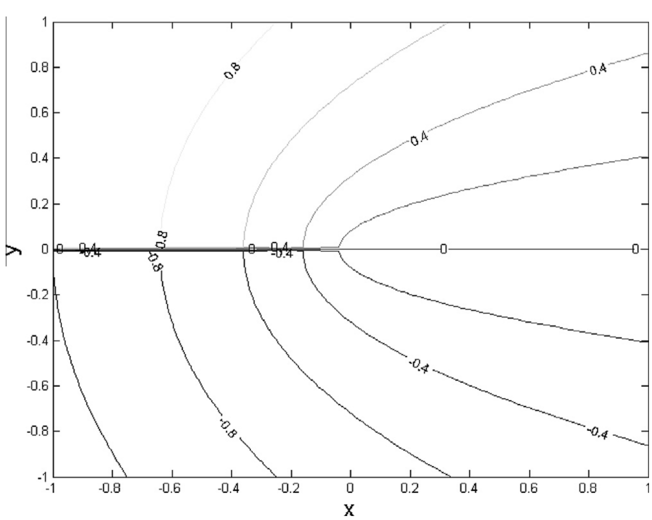

Fig. C.2. Enrichment function $\gamma_2 = \sqrt{r}\sin(\theta/2)$.

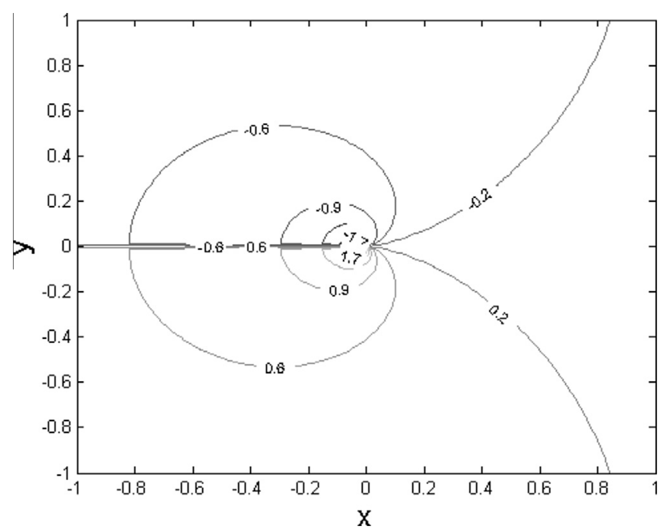

Fig. C.4. Derivative of enrichment function $\frac{d\gamma_1}{dy}$.

2. Literature review

2.1. Thermal stresses surrounding a crack tip

The gradient of displacement and temperature are both singular at the crack tip. Analytical solutions exist for the thermal and stress fields in plates with central cracks. The singular behaviour of the temperature gradient in the vicinity of a macrocrack tip is analysed in Tzou (1990, 1992) and a term analogous to the stress intensity factor is presented. This is the called the intensity factor of thermal gradient and gives an indication of the power of the singularity in heat flux at the crack tip. This intensity of heat flux at the crack tip causes high thermal stresses and can cause crack propogation. In Sumi and Katayama (1980), the thermal stresses are derived by the complex variable method. Thermal stress fields near the tip of a mode I crack in a functionally graded material are derived based on the method of displacement potentials in Kidane et al. (2008). It is observed that the temperature field disturbed by the crack influences the maximum shear stress. In Choi (2011), a method of Fourier integral transform is used in conjunction with the coordinate transformations of field variables in the basic thermoelasticity equations. Heat flux and stress singularities are analysed for different orientations of crack and combinations of material and geometric parameters. The paper addresses the fact that when crack closure occurs due to heating, the frictional contact of the crack faces would invalidate the traction free boundary condition. Also, a partially insulated crack face condition is considered to see what effect this has on stress. The thermal stress is shown to be less severe when the crack is partially insulated compared to a fully insulated crack. Results obtained for a fully insulated crack were validated with a closed form solution (Sekine, 1977) and gave exact agreement.

Elastic crack tip stress fields can be derived from the complex stress potentials of Muskhelishvili (1977), leading to the well known solution of Williams (1957). The solution is an expansion in powers of $r^{m/2}$ for integers m, so taking the derivative gives a singularity of $r^{-1/2}$ at $r = 0$. The mathematical crack tip, $r = 0$, is non physical because in reality plasticity would blunt the sharp crack tip. However, with the small scale yielding assumption, the asymptotic form of the crack tip field is suitable beyond a small radius surrounding the crack tip.